

## Normalization

Goal = BCNF = Boyce-Codd Normal Form = all FD's follow from the fact “key  $\rightarrow$  everything.”

- Formally,  $R$  is in BCNF if every nontrivial FD for  $R$ , say  $X \rightarrow A$ , has  $X$  a superkey.
  - ❖ “Nontrivial” = right-side attribute not in left side.

## Why?

1. Guarantees no redundancy due to FD's.
2. Guarantees no *update anomalies* = one occurrence of a fact is updated, not all.
3. Guarantees no *deletion anomalies* = valid fact is lost when tuple is deleted.

## Example of Problems

Drinkers(name, addr, beersLiked, manf, favoriteBeer)

name	addr	beersLiked	manf	favoriteBeer
Janeway	Voyager	Bud	A.B.	WickedAle
Janeway	???	WickedAle	Pete's	???
Spock	Enterprise	Bud	???	Bud

FD's:

1. name  $\rightarrow$  addr
  2. name  $\rightarrow$  favoriteBeer
  3. beersLiked  $\rightarrow$  manf
- ???'s are redundant, since we can figure them out from the FD's.
  - Update anomalies: If Janeway gets transferred to the *Intrepid*, will we remember to change addr in each of her tuples?
  - Deletion anomalies: If nobody likes Bud, we lose track of Bud's manufacturer.

Each of the given FD's is a BCNF violation:

- Key = {name, beersLiked}
  - ❖ Each of the given FD's has a left side a proper subset of the key.

### Another Example

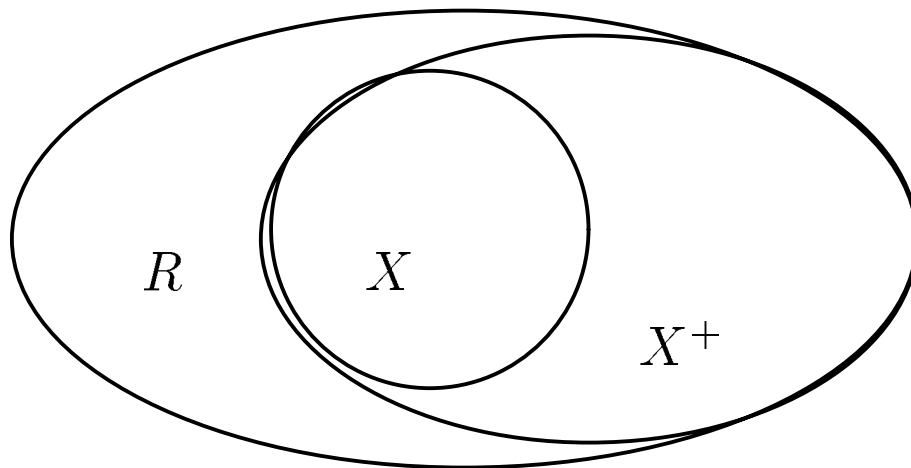
Beers(name, manf, manfAddr).

- FD's =  $\text{name} \rightarrow \text{manf}$ ,  $\text{manf} \rightarrow \text{manfAddr}$ .
- Only key is name.
  - ❖  $\text{manf} \rightarrow \text{manfAddr}$  violates BCNF with a left side unrelated to any key.

## Decomposition to Reach BCNF

Setting: relation  $R$ , given FD's  $F$ . Suppose relation  $R$  has BCNF violation  $X \rightarrow B$ .

- We need only look among FD's of  $F$  for a BCNF violation, not those that follow from  $F$ . Why?
1. Compute  $X^+$ .
    - ❖ Cannot be all attributes — why?
  2. Decompose  $R$  into  $X^+$  and  $(R - X^+) \cup X$ .



3. Find the FD's for the decomposed relations.
  - ❖ Project the FD's from  $F$  = calculate all consequents of  $F$  that involve only attributes from  $X^+$  or only from  $(R - X^+) \cup X$ .

## Example

$R = \text{Drinkers}(\underline{\text{name}}, \text{addr}, \underline{\text{beersLiked}}, \text{manf}, \text{favoriteBeer})$

$F =$

1.  $\text{name} \rightarrow \text{addr}$
2.  $\text{name} \rightarrow \text{favoriteBeer}$
3.  $\text{beersLiked} \rightarrow \text{manf}$

Pick BCNF violation  $\text{name} \rightarrow \text{addr}$ .

- Close the left side:  $\text{name}^+ = \text{name addr favoriteBeer}$ .
- Decomposed relations:  
 $\text{Drinkers1}(\underline{\text{name}}, \text{addr}, \text{favoriteBeer})$   
 $\text{Drinkers2}(\underline{\text{name}}, \underline{\text{beersLiked}}, \text{manf})$
- Projected FD's (skipping a lot of work that leads nowhere interesting):
  - ❖ For  $\text{Drinkers1}$ :  $\text{name} \rightarrow \text{addr}$  and  $\text{name} \rightarrow \text{favoriteBeer}$ .
  - ❖ For  $\text{Drinkers2}$ :  $\text{beersLiked} \rightarrow \text{manf}$ .

- BCNF violations?
  - ❖ For Drinkers1, name is key and all left sides of FD's are superkeys.
  - ❖ For Drinkers2, {name, beersLiked} is the key, and  $\text{beersLiked} \rightarrow \text{manf}$  violates BCNF.

## Decompose Drinkers2

- Close  $\text{beersLiked}^+ = \text{beersLiked}, \text{manf}$ .
- Decompose:
  - Drinkers3(beersLiked, manf)
  - Drinkers4(name, beersLiked)
- Resulting relations are all in BCNF:
  - Drinkers1(name, addr, favoriteBeer)
  - Drinkers3(beersLiked, manf)
  - Drinkers4(name, beersLiked)

## 3NF

One FD structure causes problems:

- If you decompose, you can't check the FD's in the decomposed relations.
- If you don't decompose, you violate BCNF.

Abstractly:  $AB \rightarrow C$  and  $C \rightarrow B$ .

- In book: `title city`  $\rightarrow$  `theatre` and `theatre`  $\rightarrow$  `city`.
- Another example: `street city`  $\rightarrow$  `zip`, `zip`  $\rightarrow$  `city`.

Keys:  $\{A, B\}$  and  $\{A, C\}$ , but  $C \rightarrow B$  has a left side not a superkey.

- Suggests decomposition into  $BC$  and  $AC$ .
  - ❖ But you can't check the FD  $AB \rightarrow C$  in these relations.

## Example

$A = \text{street}$ ,  $B = \text{city}$ ,  $C = \text{zip}$ .

street	zip
545 Tech Sq.	02138
545 Tech Sq.	02139

city	zip
Cambridge	02138
Cambridge	02139

Join:

city	street	zip
Cambridge	545 Tech Sq.	02138
Cambridge	545 Tech Sq.	02139



## “Elegant” Workaround

Define the problem away.

- A relation  $R$  is in 3NF iff for every nontrivial FD  $X \rightarrow A$ , either:
  1.  $X$  is a superkey, or
  2.  $A$  is *prime* = member of at least one key.
- Thus, the canonical problem goes away: you don't have to decompose because all attributes are prime.

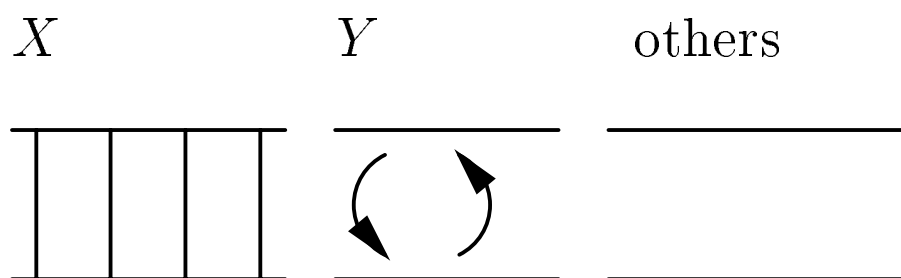
## What 3NF Gives You

There are two important properties of a decomposition:

1. We should be able to recover from the decomposed relations the data of the original.
  - ❖ Recovery involves projection and join, which we shall defer until we've discussed relational algebra.
2. We should be able to check that the FD's for the original relation are satisfied by checking the projections of those FD's in the decomposed relations.
  - Without proof, we assert that it is always possible to decompose into BCNF and satisfy (1).
  - Also without proof, we can decompose into 3NF and satisfy both (1) and (2).
  - But it is not possible to decompose into BNCF and get both (1) and (2).
    - ❖ Street-city-zip is an example of this point.

## Multivalued Dependencies

The *multivalued dependency*  $X \twoheadrightarrow Y$  holds in a relation  $R$  if whenever we have two tuples of  $R$  that agree in all the attributes of  $X$ , then we can swap their  $Y$  components and get two new tuples that are also in  $R$ .



## Example

Drinkers(name, addr, phones, beersLiked) with MVD  $\text{name} \twoheadrightarrow \text{phones}$ . If Drinkers has the two tuples:

name	addr	phones	beersLiked
sue	<i>a</i>	<i>p1</i>	<i>b1</i>
sue	<i>a</i>	<i>p2</i>	<i>b2</i>

it must also have the same tuples with **phones** components swapped:

name	addr	phones	beersLiked
sue	<i>a</i>	<i>p1</i>	<i>b2</i>
sue	<i>a</i>	<i>p2</i>	<i>b1</i>

- Note: we must check this condition for *all* pairs of tuples that agree on **name**, not just one pair.

## MVD Rules

1. Every FD is an MVD.
  - ❖ Because if  $X \rightarrow Y$ , then swapping  $Y$ 's between tuples that agree on  $X$  doesn't create new tuples.
  - ❖ Example, in `Drinkers`:  $\text{name} \twoheadrightarrow \text{addr}$ .
2. *Complementation*: if  $X \twoheadrightarrow Y$ , then  $X \twoheadrightarrow Z$ , where  $Z$  is all attributes not in  $X$  or  $Y$ .
  - ❖ Example: since  $\text{name} \twoheadrightarrow \text{phones}$  holds in `Drinkers`, so does  $\text{name} \twoheadrightarrow \text{addr beersLiked}$ .

## Splitting Doesn't Hold

Sometimes you need to have several attributes on the right of an MVD. For example:

Drinkers(name, areaCode, phones, beersLiked, beerManf)

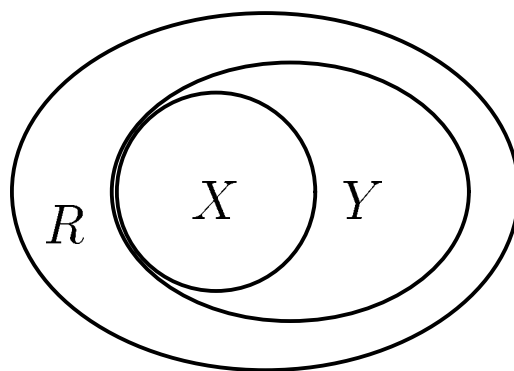
name	areaCode	phones	BeersLiked	beerManf
Sue	650	555-1111	Bud	A.B.
Sue	650	555-1111	WickedAle	Pete's
Sue	415	555-9999	Bud	A.B.
Sue	415	555-9999	WickedAle	Pete's

- name  $\twoheadrightarrow$  areaCode phones holds, but neither name  $\twoheadrightarrow$  areaCode nor name  $\twoheadrightarrow$  phones do.

## 4NF

Eliminate redundancy due to multiplicative effect of MVD's.

- Roughly: treat MVD's as FD's for decomposition, but not for finding keys.
- Formally:  $R$  is in Fourth Normal Form if whenever MVD  $X \twoheadrightarrow Y$  is *nontrivial* ( $Y$  is not a subset of  $X$ , and  $X \cup Y$  is not all attributes), then  $X$  is a superkey.
  - ❖ Remember,  $X \rightarrow Y$  implies  $X \twoheadrightarrow Y$ , so 4NF is more stringent than BCNF.
- Decompose  $R$ , using 4NF violation  $X \twoheadrightarrow Y$ , into  $XY$  and  $X \cup (R - Y)$ .



## Example

Drinkers(name, addr, phones, beersLiked)

- FD:  $\text{name} \rightarrow \text{addr}$
- Nontrivial MVD's:  $\text{name} \twoheadrightarrow \text{phones}$  and  $\text{name} \twoheadrightarrow \text{beersLiked}$ .
- Only key:  $\{\text{name}, \text{phones}, \text{beersLiked}\}$
- All three dependencies above violate 4NF.
- Successive decomposition yields 4NF relations:  
    D1(name, addr)  
    D2(name, phones)  
    D3(name, beersLiked)