More Clustering

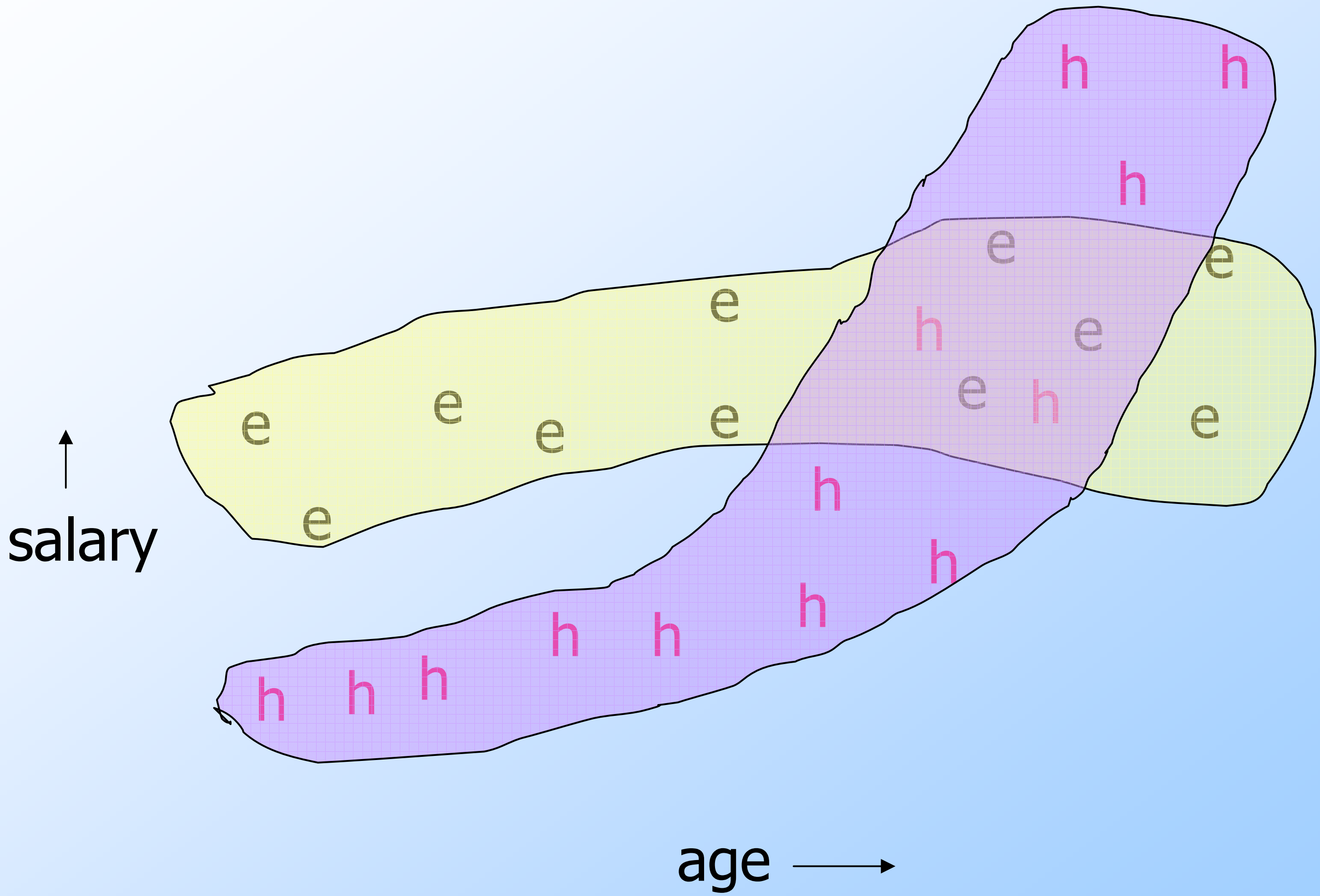
CURE Algorithm

Clustering Streams

The CURE Algorithm

- Problem with BFR/*k* -means:
  - Assumes clusters are normally distributed in each dimension.
  - And axes are fixed --- ellipses at an angle are *not* OK.
- CURE:
  - Assumes a Euclidean distance.
  - Allows clusters to assume any shape.

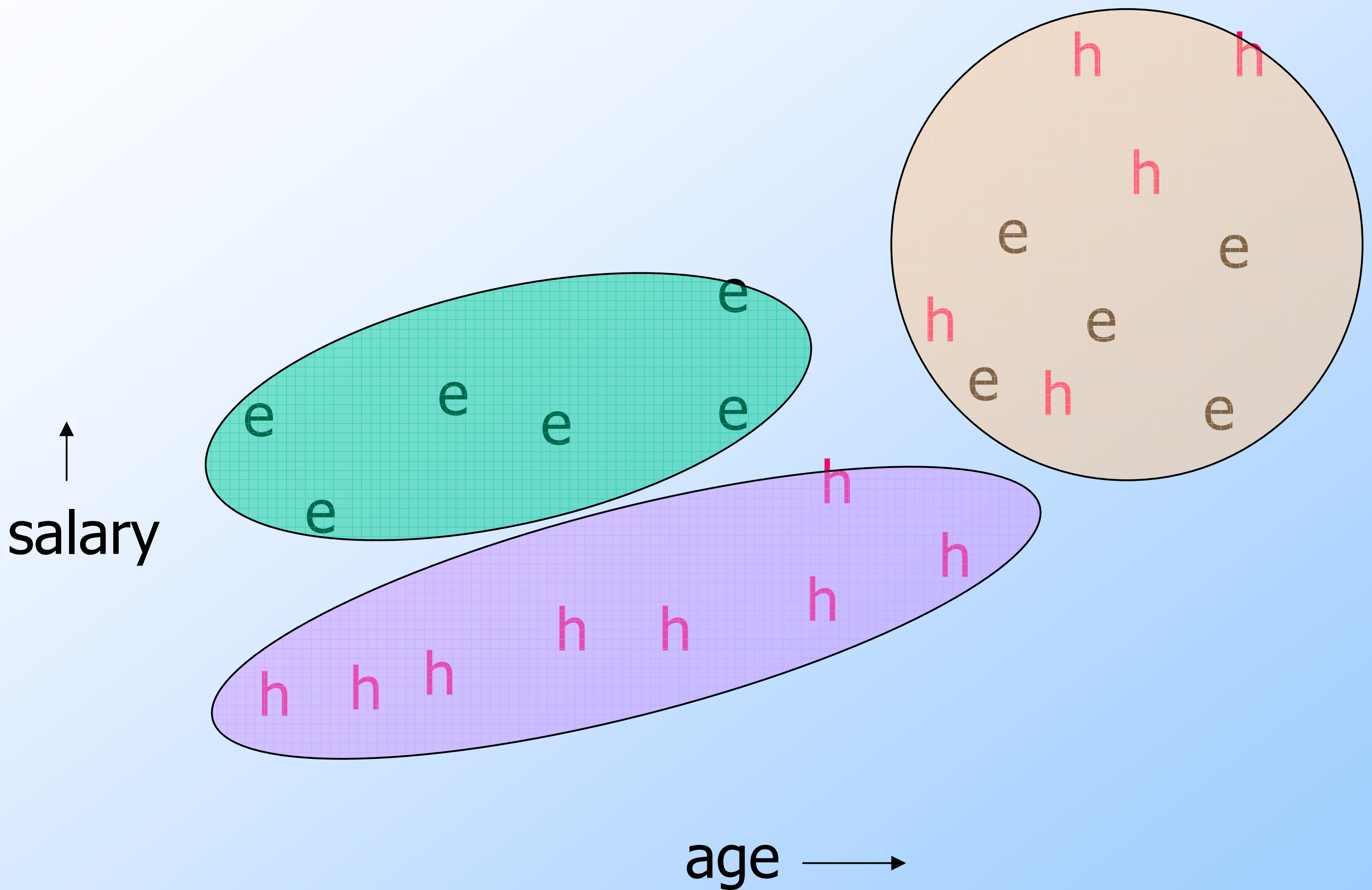
Example: Stanford Faculty Salaries



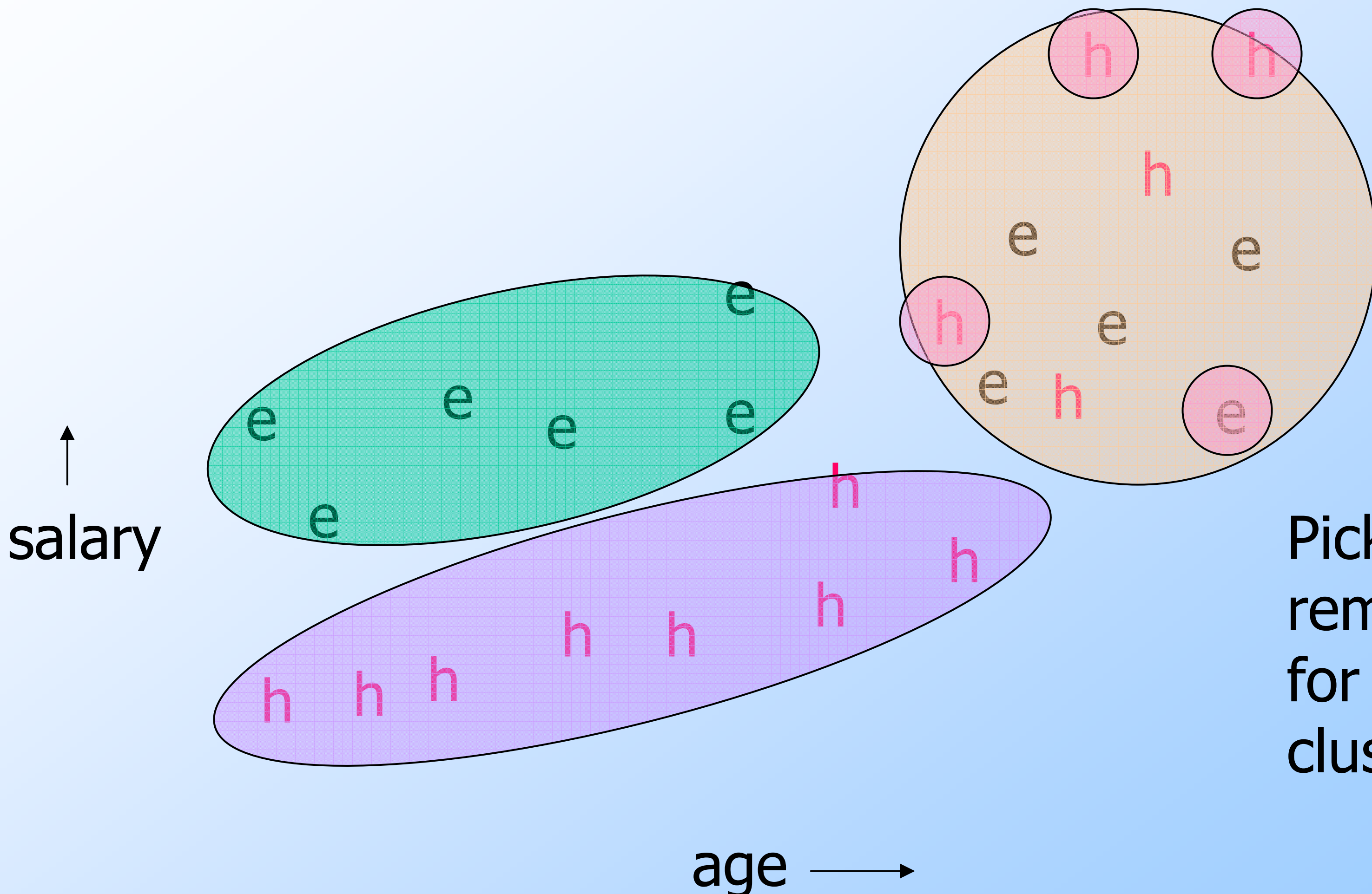
Starting CURE

1. Pick a random sample of points that fit in main memory.
2. Cluster these points hierarchically --- group nearest points/clusters.
3. For each cluster, pick a sample of points, as dispersed as possible.
4. From the sample, pick representatives by moving them (say) 20% toward the centroid of the cluster.

Example: Initial Clusters

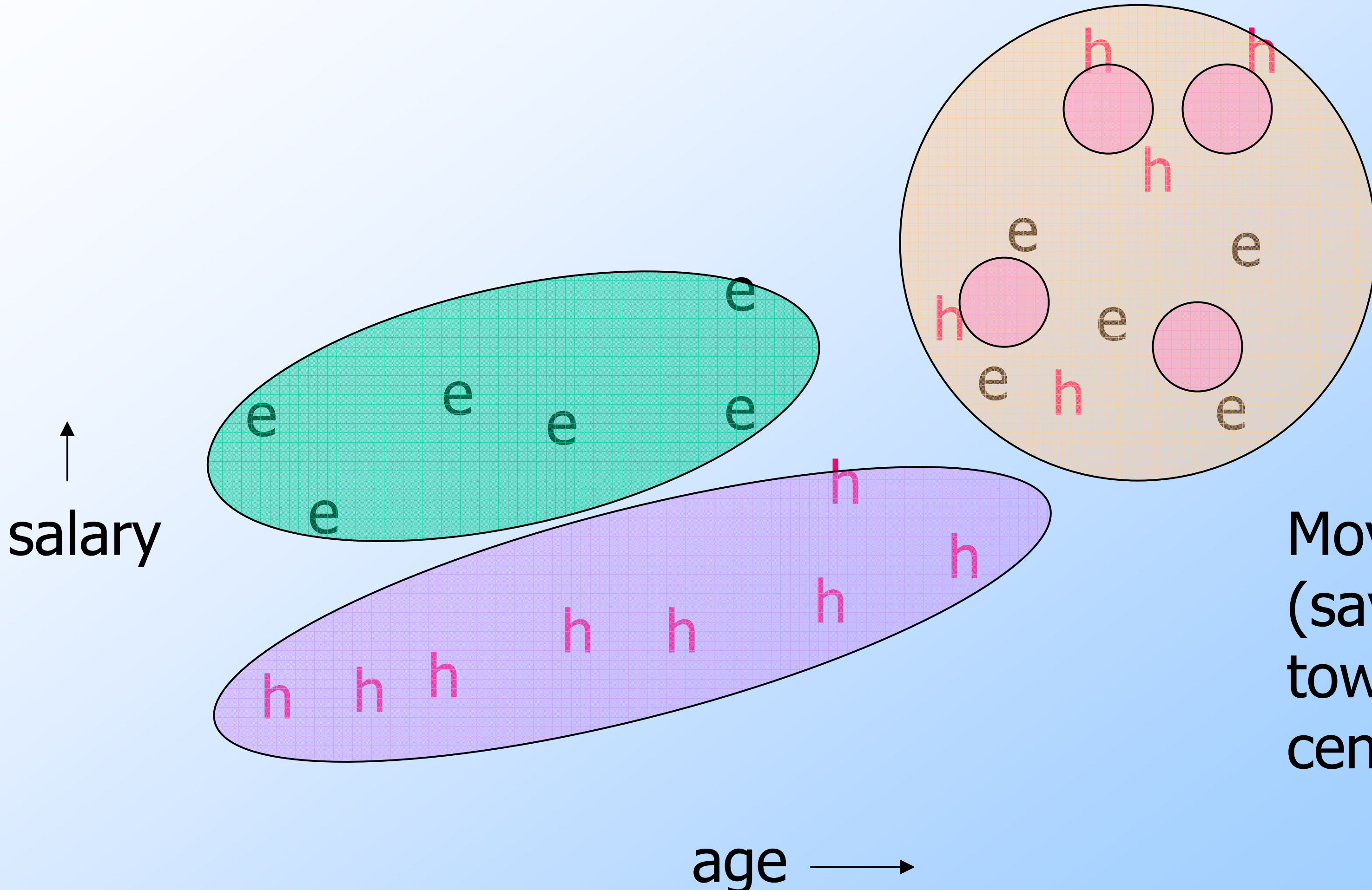


Example: Pick Dispersed Points



Pick (say) 4 remote points for each cluster.

Example: Pick Dispersed Points



Move points (say) 20% toward the centroid.

Finishing CURE

- Now, visit each point p in the data set.
- Place it in the "closest cluster."
  - Normal definition of "closest": that cluster with the closest (to p) among all the sample points of all the clusters.

Clustering a Stream (New Topic)

- Assume points enter in a stream.
- Maintain a sliding window of points.
- Queries ask for clusters of points within some suffix of the window.
- Only important issue: where are the cluster centroids.
  - There is no notion of "all the points" in a stream.

BDMO Approach

- BDMO = Babcock, Datar, Motwani, O'Callaghan.
- k –means based.
- Can use less than O(N) space for windows of size N.
- Generalizes trick of DGIM: buckets of increasing "weight."

Recall DGIM

- Maintains a sequence of buckets B_1, B_2, …
- Buckets have timestamps (most recent stream element in bucket).
- Sizes of buckets nondecreasing.
  - In DGIM size = power of 2.
- Either 1 or 2 of each size.

Alternative Combining Rule

- Instead of "combine the 2nd and 3rd of any one size" we could say:
- "Combine B_{i+1} and B_i if $size(B_{i+1} \cup B_i) < size(B_{i-1} \cup B_{i-2} \cup \ldots \cup B_1)$."
  - If B_{i+1}, B_i, and B_{i-1} are the same size, inequality must hold (almost).
  - If B_{i-1} is smaller, it cannot hold.

Buckets for Clustering

- In place of "size" (number of 1's) we use (an approximation to) the sum of the distances from all points to the centroid of their cluster.
- Merge consecutive buckets if the "size" of the merged bucket is less than the sum of the sizes of all later buckets.

Consequence of Merge Rule

- In a stable list of buckets, any two consecutive buckets are "bigger" than all smaller buckets.
- Thus, "sizes" grow exponentially.
- If there is a limit on total "size," then the number of buckets is O(log *N*).
    - *N* = window size.
  - E.g., all points are in a fixed hypercube.

Outline of Algorithm

1. What do buckets look like?
   - Clusters at various levels, represented by centroids.
2. How do we merge buckets?
   - Keep # of clusters at each level small.
3. What happens when we query?
   - Final clustering of all clusters of all relevant buckets.

Organization of Buckets

- Each bucket consists of clusters at some number of levels.
  - 4 levels in our examples.
- Clusters represented by:
  1. Location of centroid.
  2. *Weight* = number of points in the cluster.
  3. *Cost* = upper bound on sum of distances from member points to centroid.

Processing Buckets --- (1)

- Actions determined by N (window size) and k (desired number of clusters).
- Also uses a tuning parameter τ for which we use 1/4 to simplify.
  - $1/\tau$ is the number of levels of clusters.

Processing Buckets --- (2)

- Initialize a new bucket with k new points.
  - Each is a cluster at level 0.
- If the timestamp of the oldest bucket is outside the window, delete that bucket.

Level-0 Clusters

- A single point *p* is represented by (p, 1, 0).
- That is:
  1. A point is its own centroid.
  2. The cluster has one point.
  3. The sum of distances to the centroid is 0.

Merging Buckets --- (1)

- Needed in two situations:
  1. We have to process a query, which requires us to (temporarily) merge some tail of the bucket sequence.
  2. We have just added a new (most recent) bucket and we need to check the rule about two consecutive buckets being "bigger" than all that follow.

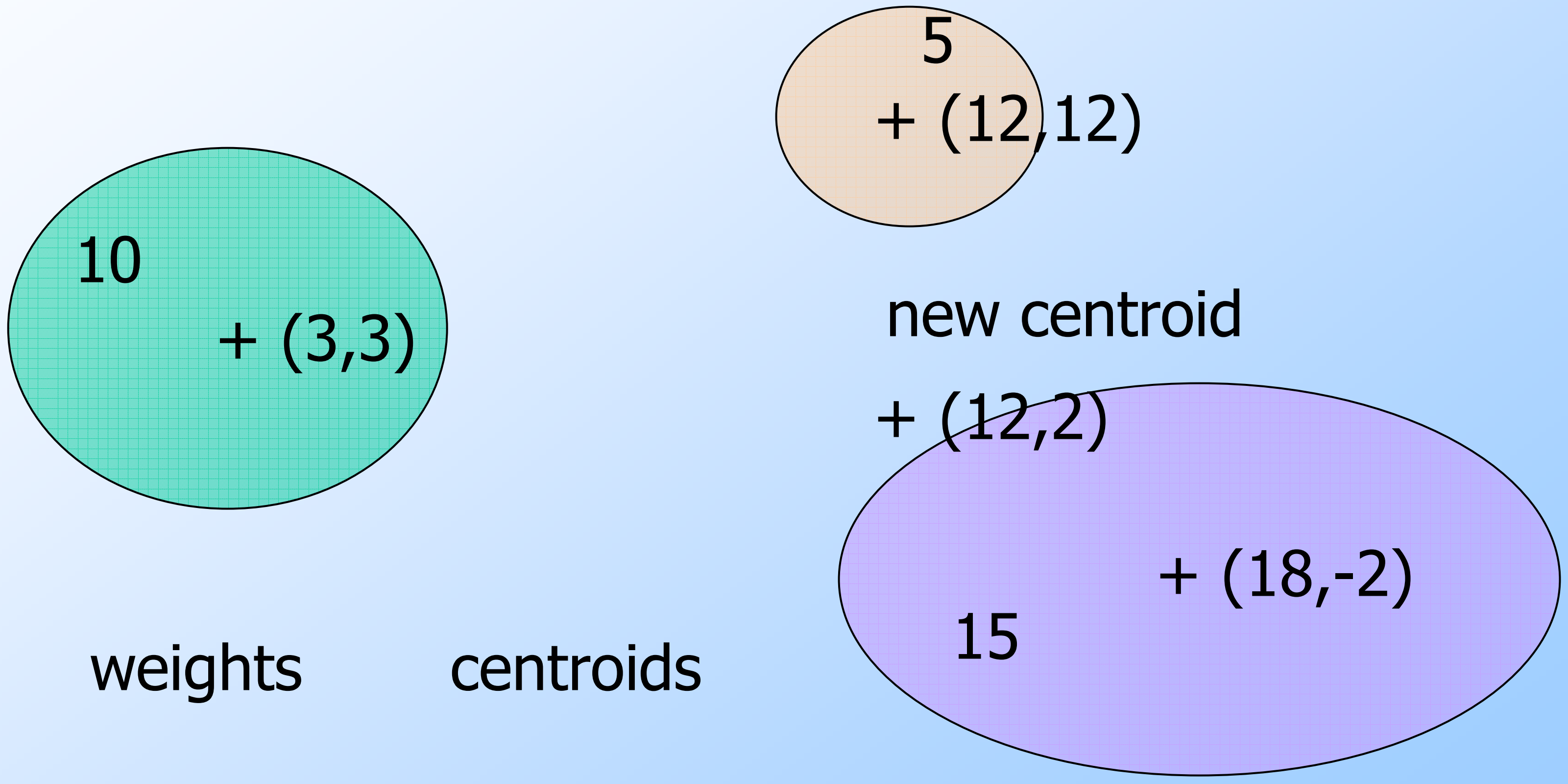
Merging Buckets --- (2)

- Step 1: Take the union of the clusters at each level.
- Step 2: If the number of clusters (points) at level 0 is now more than $N^{1/4}$, cluster them into k clusters.
  - These become clusters at level 1.
- Steps 3,…: Repeat, going up the levels, if needed.

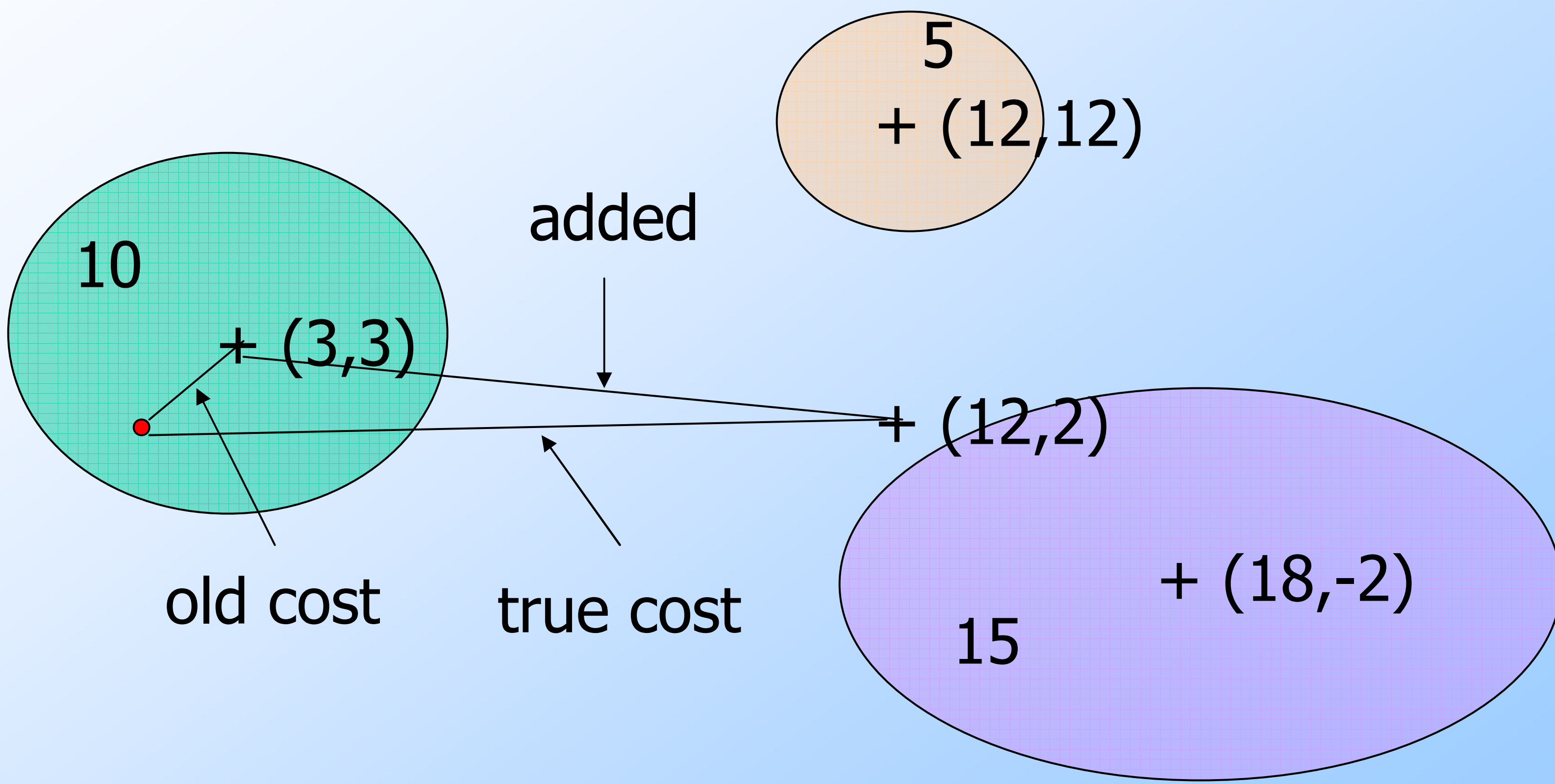
Representing New Clusters

- Centroid = weighted average of centroids of component clusters.
- Weight = sum of weights.
- Cost = sum over all component clusters of:
  1. Cost of component cluster.
  2. Weight of component times distance from its centroid to new centroid.

Example: New Centroid



Example: New Costs



Queries

- Find all the buckets within the range of the query.
  - The last bucket may be only partially within the range.
- Cluster all clusters at all levels into *k* clusters.
- Return the *k* centroids.

Error in Estimation

- Goal is to pick the k centroids that minimize the *true* cost (sum of distances from each point to its centroid).
- Since recorded "costs" are inexact, there can be a factor of 2 error at each level.
- Additional error because some of last bucket may not belong.
  - But fraction of spurious points is small (why?).

Effect of Cost-Errors

1. Alter when buckets get combined.
   - Not really important.
2. Produce suboptimal clustering at any stage of the algorithm.
   - The real measure of how bad the output is.

Speedup of Algorithm

- As given, algorithm is slow.
  - Each new bucket causes O(log N) bucket-merger problems.
- A faster version allows the first bucket to have not k, but $N^{1/2}$ (or in general $N^{2\tau}$) points.
  - A number of consequences, including slower queries, more space.