CS109A Notes for Lecture 2/26/96

Rooted Trees

Collection of *nodes*, one of which is the *root*.

- Nodes $\neq$ root have unique *parent* node.
- Each nonroot can reach the root by following parent links one or more times.

Important Definitions

- If node p is the parent of node c, then c is a *child* of p.
- *Leaf*: no children; *interior node* has children.
- *Path* = list of nodes $(m_1, m_2, \ldots, m_k)$ such that each is the parent of the following node.
  - ☐ It is "from m_1 to m_k."
  - ☐ *Length* of the path $= k - 1$, the number of links, not nodes.
- If there is a path from m to n, then m is an *ancestor* of n and n is a *descendant* of m.
  - ☐ Note $m = n$ is possible.
  - ☐ *Proper* ancestors, descendants exclude the possibility $m = n$.
- *Height* of a node n is the length of the longest path from n to a leaf.
  - ☐ *Height* of a tree is the height of its root.
- *Depth* of a node n is the length of the path from the root to n.
- The *subtree rooted at* node n is all the descendants of n (including n, of course!).
- The children of a given node are often *ordered* "from the left."
  - ☐ If so, and child c_1 is to the left of child c_2, then all nodes in the subtree rooted at c_1 are said to be "to the left" of those in the subtree of c_2.

- Nodes may have *labels*, which are values associated with the nodes.

Example: *Expression trees*: Labels are operands or operators.

- Leaf = operand; interior node = operator.
- Children are roots of the subexpressions to which the operator is applied.

Leftmost-Child, Right-Sibling Tree Representation

Each node has a pointer to

1. Its leftmost child.
2. Its *right-sibling* = node immediately to the right having the same parent.

- Advantage: represents trees without limit on number of children.
- Disadvantage: to find ith child of node n you must traverse list of right-sibling pointers starting at the leftmost child of n.
    - □ Nevertheless, this representation is the preferred approach in most cases.

Recursions on Trees

Many algorithms to process trees are designed with a basis = leaves and induction = interior nodes.

Example: Expressions in:

- *infix* (common form — operator between operands),
- *prefix* (operator before operands, like function calls without parentheses),
- *postfix* (operator after operands — important for compilers, because it gives the order in which computer must do things).

A recursive algorithm to convert from infix expression trees to postfix:

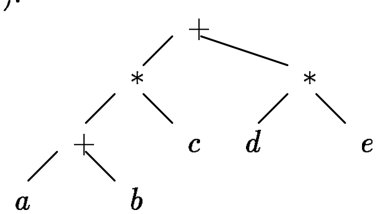
Basis: For a leaf, just print the operand.

Induction: At an interior node, having an operator:

- For each child, in order from the left, apply the algorithm at the child.
- Finally, list the operator.

Example: : Infix expression $(a+b)*c+(d*e)$.

- Expression tree (note parentheses are not needed):



Result of Recursive algorithm: $ab+c*de*+$.

Preorder, Postorder Traversals

Two common ways to explore a tree.

- Assume some "action" is to be taken at each node, e.g. printing its label.
- Postorder:

Basis: Visit a leaf by performing the action there.

Induction: Visit an interior node by visiting all its children, from the left, then performing the action at the node.

Example: If the action is to list the label, postorder traversal converts the example expression tree to its equivalent postfix expression.

- Preorder:

Basis: Visit a leaf by performing the action there.

Induction: Perform the action at the node, then visit its children, from the left.

Example: If the action is to print labels, the result of a preorder traversal of or previous example tree is $+*+abc*de$.

Structural Induction

- Basis = leaves (one-node trees).
- Induction = interior nodes (trees with $\geq$ 2 nodes). Assume the statement holds for the subtrees at the children of the root and prove the statement for the whole tree.
- A shorthand for induction on height of a tree or number of nodes in a tree.

Example: Consider the LMC-RS (leftmost-child, right-sibling) data structure for trees.

- $S(T)$: T has one more `NULL` pointer than nodes.

Basis: T consists of a single node. It has neither a LMC nor a RS, so 2 `NULL` pointers and 1 node. Hence the basis holds.

Induction: T has a root r and one or more subtrees $T_1, T_2, \ldots, T_k$. By the inductive hypothesis, each of these k trees *by itself* has one more `NULL` than node.

  - ☐ Think: "excess" is k.

- When we include the root, we add one node and one `NULL` pointer (the root's LMC).

  - ☐ Excess is still k.

- However, when they are children of a common node, the LMC pointers of the first $k-1$ become non-`NULL` .

  - ☐ Excess is reduced to 1, proving $S(T)$.