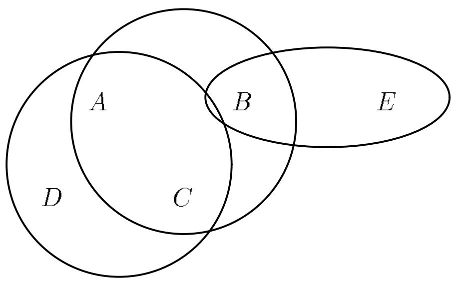
Hypergraphs

Hypergraph = nodes plus (hyper)edges that are sets of any number of nodes.

- Applications include optimizing queries that are joins and representing "universal relations" (a useful data-modeling concept).
- Typically, nodes represent attributes and hyperedges are sets of attributes.

Example

Suppose we have relations with schemas ABC, ACD, and BE. This database schema could be represented by the hypergraph



Acyclic Hypergraphs

These have some useful properties that make query optimization easier than the general case. Most "natural" queries correspond to acyclic hypergraphs.

Definition depends on *GYO reduction*; GYO = Graham-Yu-Ozsoyoglu.

- An *ear* is a hyperedge H such that we can divide its nodes into two groups: those that appear in H and no other hyperedge and those that are contained in another hyperedge G.
  - Note that an isolated edge is an ear; no G is needed.

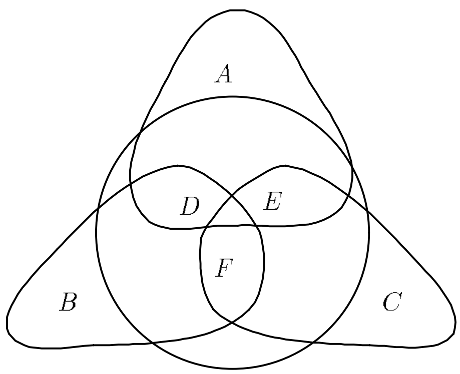
- GYO reduction of a hypergraph is the process of repeatedly finding ears and removing them. That is, we remove those nodes that are in the ear and no other hyperedge; then we remove the hyperedge itself, leaving the other nodes.
  - We say that ear H is *consumed* by G, if all the nodes that are not unique to H are in G.
  - If a hypergraph is reduced to nothing

by GYO reduction, then it is said to be *acyclic*.

- Aside: "acyclic" makes sense: if the hypergraph is an ordinary graph, it is acyclic iff it is a tree.

Example

Here is an acyclic hypergraph



- The central hyperedge DEF can consume each of the other three hyperedges.
- At that time, the remaining hyperedge is trivially an ear, since all of its nodes are unique to it.

Formal GYO Reduction

The original definition of GYO reduction consisted of the following two steps:

1. Eliminate a node that is in only one hyperedge.
2. Delete a hyperedge that is contained in another.

The goal is to reduce a hypergraph to a single, empty hyperedge.

- You need to look at GYO reduction this way to show that there is a unique GYO reduction of any hypergraph, acyclic or not.
    - Key idea of proof: candidates for step (1) remain candidates, no matter what other steps are taken.

Dangling Tuple Elimination

- Useful as a first step in optimizing large joins.

- A collection of relations $R_1, R_2, \ldots, R_n$ is *locally join consistent* if for each i and j there are no tuples that dangle between R_i and R_j. Formally: $\pi_{R_i}(R_i \bowtie R_j) = R_i$, and similarly when i and j are reversed.

- These relations are *globally join consistent* if there are no dangling tuples when considered as a group. Formally, for all i:

  $$\pi_{R_i}(R_1 \bowtie R_2 \bowtie \cdots \bowtie R_n) = R_i$$

- Easy to check global consistency implies local consistently.

  - What about the opposite?

Theorem

If the relation schemas $R_1, R_2, \ldots, R_n$ form an acyclic hypergraph, then whenever relations for these schemas are locally consistent, they are globally consistent.

Proof

Induction on n, the number of hyperedges (relations in the join).

Basis: For $n = 1$ there is nothing to check.

Induction: Assume for $n - 1$ hyperedges, and prove for n.

- Let E be the first ear in a GYO reduction, and let G be the remaining hypergraph.

- Since G has local consistency and $n - 1$ hyperedges, by the inductive hypothesis, G is globally consistent.

  - That is, every tuple of every relation of G appears in the result of the join.

- E was consumed by some hyperedge H, and E is locally consistent with H. Therefore, each tuple t of E joins with some tuple s of H.

- s appears as part of some tuple r in the join of the relations in G. Since attributes of E are either unique to it, or in H, t joins with r.

  - Thus, t particpates in the join of all n relations.

- However, if the hypergraph is not acyclic, we can always find relations that are locally consistent but not globally consistent.

Example

Consider $AB = \{00, 11\}$, $BC = \{00, 11\}$, and $AC = \{01, 10\}$.

- Any two relations are join-consistent. E.g., $AB \bowtie AC = \{001, 110\}$, which projected onto AB is $\{00, 11\}$.
- But $AB \bowtie BC \bowtie AC = \emptyset$, so the relations are not globally consistent.

Reduction by Semijoins

If we are to take the join of several relations, it is often efficient to first remove the dangling tuples.

- It guarantees that whatever order we join in, the result never shrinks. Thus, the total work is proportional to the output, and we can't do more than a constant factor better than that.
- To reduce relations to globally consistent subsets, we can use the *semijoin* operation:

$$R := R \ltimes S = \pi_R(R \bowtie S) = R \bowtie \left(\pi_R(S)\right)$$

- Sometimes, semijoins don't help eliminating dangling tuples.
    - ❖ For example, AB, BC, and AC above are not changed by any semijoin.

- However, if the hypergraph is acyclic, the following algorithm produces a *full reducer* for a set of relations.
    - ❖ That is, the result is a set of globally join-consistent relations.

1. Pick an ear E that can be consumed by hyperedge H. Execute the semijoin $H := H \ltimes E$.
2. Recursively generate a full reducer for the hypergraph with E removed.
3. Append the semijoin $E := E \ltimes H$.

Example

Consider the relation schemas ABC, ACD, and DE.

- ACD is an ear that is consumed by ABC.
- In the resulting hypergraph, ABC can be consumed by BE.
- The full reducer:

ABC := $ABC \ltimes ACD$
BE := $BE \ltimes ABC$
ABC := $ABC \ltimes BE$
ACD := $ACD \ltimes ABC$

Proof It Works

- After step (1), it is impossible for the join of the remaining hyperedges to have a tuple that doesn't join with any tuple of E.
- Inductively, step (2) leaves the relations other than E in a globally join-consistent state.
- Then, step (3) eliminates from E any tuples that do not join with the other relations.