CS109B Notes for Lecture 5/12/95

Notational Shift

Instead of the combersome `AND`, `OR`, etc., it is more efficient to think of `AND` as product and `OR` as sum.

- Also, `NOT` applied to a variable p can be denoted with a bar, as $\bar{p}$.
- And of course `TRUE` and `FALSE` have shorthands 1 and 0, respectively.

Example: (p`OR NOT`q)`AND`(q`OR NOT`p) is better expressed as $(p + \bar{q})(q + \bar{p})$.

Logical Expressions From Truth Tables

There is a mechanical way to get an expression from a truth table.

1. For each row that has value 1, construct the *minterm*, that is, the product (`AND`) of one "literal" for each variable.
   - A *literal* is either a variable p or a negated variable, $\bar{p}$.
   - For each row, pick p if there is a 1 in the column for p and pick $\bar{p}$ if there is a 0 in the column for p.
2. The expression is the `OR` of all the minterms.

Example:

p	q	$f(p,q)$
0	0	1
0	1	0
1	0	1
1	1	1

- There are three rows with value 1: 00, 10, and 11.
- Their minterms are $\bar{p}\bar{q}$, $p\bar{q}$, and pq, respectively.
- Thus, an expression for this function is $\bar{p}\bar{q} + p\bar{q} + pq$,

- Simpler equivalent expressions are $q \rightarrow p$ and $\bar{q} + p$.

Sums of Product Form

A sum of minterms is often not the simplest expression for a logical function.

- A product of fewer literals "covers" more than one minterm.
  - A product P *covers* a product Q if whenever Q is true, P is true.
  - Equivalently, product P covers product Q iff every literal in P appears in Q.

Example: Product (of one literal) p covers products $p\bar{q}$ and pq, while product $\bar{q}$ covers $\bar{p}\bar{q}$ and $p\bar{q}$,

- That explains why $\bar{q} + p$ is equivalent to $\bar{p}\bar{q} + p\bar{q} + pq$:
  - Each product of the second is covered by at least one product of the first, yet
  - Each product of the first covers *only* minterms of the second.

Strategy For Simplifying Sums of Minterms

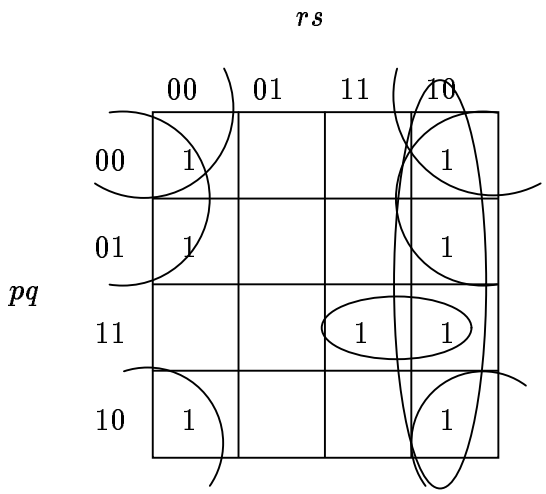
- Think of minterms as points. A logical function is a set S of points.
- Find products that cover only minterms in S.
  - These products are called *implicants* of the function.
- Choose only *prime implicants*, which are maximal implicants in the sense that if you delete any literal from the product, it now covers some point not in the set S.
- Select enough implicants that every point of S is covered by at least one implicant.

Karnaugh Maps

- A visual aid for representing functions, finding prime implicants, and covering a set of minterms.

- Works well for up to 4 variables.
  - 2, 3 variable cases in FCS; we'll look only at the 4-variable case.

Example:



- Two variables per dimension. Note order of values: 00, 01, 11, 10.
- Squares represent minterms; 1 means the represented function has value 1 for that minterm.
- Implicants are rectangles, but sides must be a power of 2, i.e., 1, 2, or 4.
- Rows and columns are adjacent in the "end-around" sense, e.g., the 00 column is next to the 10 column and similarly for the rows.
  - Note that because of row order, "adjacent" means "differs in only one variable."
  - That's why adjacent points are always covered by some common implicant and "implicant = rectangle" makes sense.

Reading Implicants From Rectangles

- If all points of a rectangle have $x = 1$, then x is part of the product.

- If all points of the rectangle have $x = 0$, then $\bar{x}$ is part of the product.
- If some points have $x = 1$ and others have $x = 0$, then neither x nor $\bar{x}$ is in the product.

Example: The implicants marked in the figure above are: $\bar{q}\bar{s}$ (the corners), $r\bar{s}$ (the right column), pqr (the pair of points circled), and $\bar{p}\bar{s}$ (the left and right top halves).

- All these implicants are prime, because they cannot be doubled in size along any dimension without taking in a point that is not part of the function.
- All these prime implicants except $r\bar{s}$ are *essential*, in the sense that if you delete any from the sum there is some uncovered point.
- Thus, $\bar{q}\bar{s} + pqr + \bar{p}\bar{s}$ is a minimal sum of products for the function represented.

Class Problem

Doodle the prime implicants of this function here:

pq \ rs	00	01	11	10
00	1			1
01		1	1	1
11		1	1	
10	1			

- Are they all essential?