Challenge Problem Set 1 - Solution

April 20, 2010

Problem 1.(5 Points)

The problem is equivalent to the problem of putting n indistinguishable objects into 3 buckets. The answer is $\binom{n+2}{2} = \frac{(n+2)(n+1)}{2}$.

Problem 2.(5 Points)

The automaton tells whether the string length is even (state A) or odd (state B), accepting in the latter case. It is an easy induction on $|w|$ to show that $\hat{\delta}(A, w) = B$ if and only if $|w|$ is odd.

Basis: $|w| = 0$. Then w, the empty string has a length of 0, which is even, and $\delta(A, w) = A$, which implies that w is not accepted.

$|w| = 1$. Then w, a string length of 1, which is odd, and $\delta(A, w) = B$, which implies that w is accepted. *Note: This step is technically not necessary but is presented here for clarity*

Induction: Assume the statement for strings shorter than w. Then $w = za$, where a is either 0 or 1.

If $|w|$ is odd : This implies that $|z|$ is even. By inductive hypothesis $\hat{\delta}(A, z) = A$. According to DFA specification, then $\hat{\delta}(A, w) = \delta(\hat{\delta}(A, z), a) = B$, for any $a(0/1)$ and hence w is accepted.

If $|w|$ is even : This implies that $|z|$ is odd. By inductive hypothesis $\hat{\delta}(A, z) = B$. According to DFA specification, then $\hat{\delta}(A, w) = \delta(\hat{\delta}(A, z), a) = A$, for any $a(0/1)$ and hence w is rejected.

Thus in the light of above statements, we have that $\hat{\delta}(A, w) = B$ if and only if $|w|$ is odd.

Error Codes

- N.E(-1) Notation Error - Most of the mistakes were related to interchanging of $\hat{\hat{\delta}}$ and δ
- L.F(-1) Lacks Formalism - The proof is not completely mathematical
- N.A.(-1) Language Wrongly specified
- B.C.(-1) Missing or wrong base case
- I.S (-1) Minor mistake in the induction step
- B.M.F.(0) Requested to be more clear, formal. No points deducted
- S.S. See Solutions

Problem 3.(5 points + 1 possible extra Credit for specifying 5 State DFA)

The 20 state DFA can be developed as follows - Each state is represented by a pair ij where i is the remainder of the reverse of the input when divided by 5, and j is the remainder when 2 to the power of the length of the input is divided by 5.

We then setup the transitions as specified in Table 1

We can then reduce this 20 state DFA using minimization to obtain a 5 state DFA as in Table 2

Table 1: 20 State DFA

Current State	0	1
*01	02	12
*02	04	24
*03	01	31
*04	03	43
11	12	22
12	14	34
13	11	41
14	13	03
21	22	32
22	24	44
23	21	01
24	23	13
31	32	42
32	34	04
33	31	11
34	33	23
41	42	02
42	44	14
43	41	21
44	43	33

Table 2: 5 State DFA

Current State	0	1
*0	0	1
1	2	3
2	3	0
3	4	2
4	1	4