Methods for High Degrees of Similarity

Index-Based Methods

Exploiting Prefixes and Suffixes

Exploiting Length

Overview

- LSH-based methods are excellent for similarity thresholds that are not too high.
  - Possibly up to 80% or 90%.
- But for similarities above that, there are other methods that are more efficient.
  - And also give exact answers.

Setting: Sets as Strings

- We'll again talk about Jaccard similarity and distance of sets.
- However, now represent sets by strings (lists of symbols):
  1. Enumerate the universal set.
  2. Represent a set by the string of its elements in sorted order.

Example: Shingles

- If the universal set is k-shingles, there is a natural lexicographic order.
- Think of each shingle as a single symbol.
- Then the 2-shingling of abcad, which is the set {ab, bc, ca, ad}, is represented by the list ab, ad, bc, ca of length 4.
- Alternative: hash shingles; order by bucket number.

Example: Words

- If we treat a document as a set of words, we could order the words alphabetically.
- Better: Order words lowest-frequency-first.
- Why? We shall index documents based on the early words in their lists.
  - Documents spread over more buckets.

Jaccard and Edit Distances

- Suppose two sets have Jaccard distance J and are represented by strings s_1 and s_2. Let the LCS of s_1 and s_2 have length C and the edit distance of s_1 and s_2 be E. Then:
  - 1-J = Jaccard similarity = C/(C+E).
  - J = E/(C+E).

Works because these strings never repeat a symbol, and symbols appear in the same order.

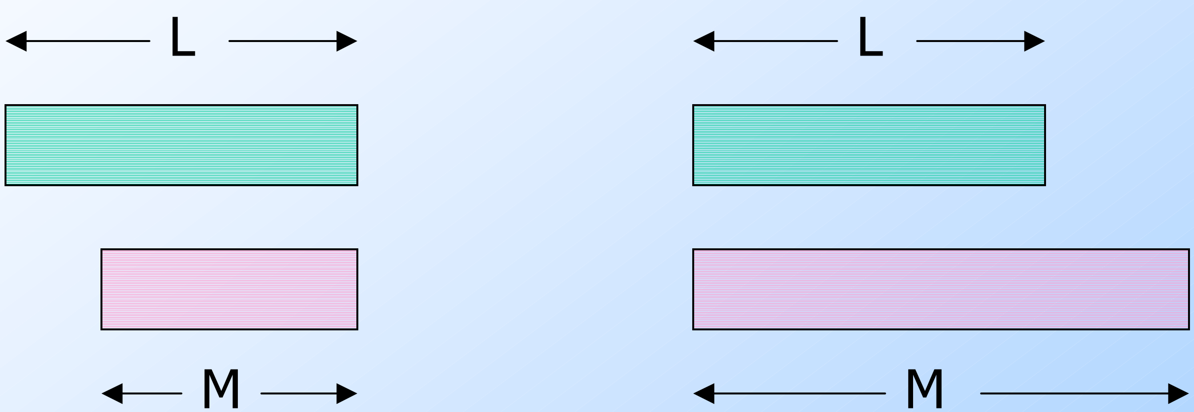
Indexes

- The general approach is to build some indexes on the set of strings.
- Then, visit each string once and use the index to find possible candidates for similarity.
- For thought: how does this approach compare with bucketizing and looking within buckets for similarity?

Length-Based Indexes

- The simplest thing to do is create an index on the length of strings.
- A string of length L can be Jaccard distance J from a string of length M only if $L \times (1-J) \leq M \leq L/(1-J)$.
- Example: if 1-J = 90% (Jaccard similarity), then M is between 90% and 111% of L.

Why the Limit on Lengths?



1-J = M/L

M = L×(1-J)

A shortest candidate

1-J = L/M

M = L/(1-J)

A longest candidate

B-Tree Indexes

- The B-tree is a perfect index structure for a length-based index.
- Given a string of length L, we can find strings in the range $L \times (1-J)$ to $L/(1-J)$ without looking at any candidates outside that range.
- But just because strings are similar in length, doesn't mean they are similar.

Aside: B-Trees

- If you didn't take CS245 yet, a B-tree is a generalization of a binary search tree, where each node has many children, and each child leads to a segment of the range of values handled by its parent.
- Typically, a node is a disk block.

Aside: B-Trees – (2)

From parent

| |50| |80| |145| |190| |225| |

Etc.

To values < 50

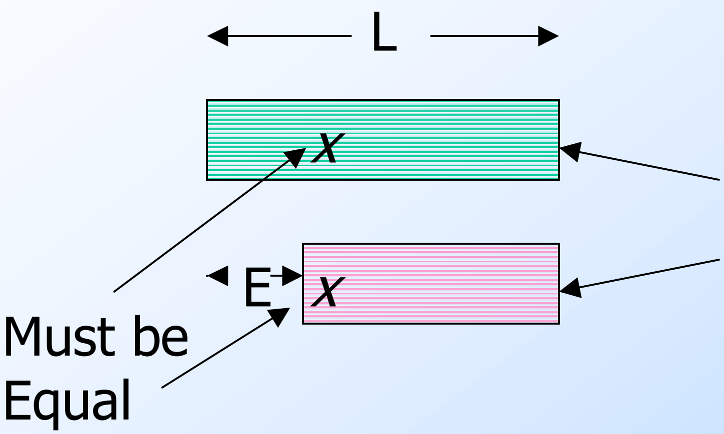
To values $\geq$ 50, < 80

To values $\geq$ 80, < 145

Prefix-Based Indexing

- If two strings are 90% similar, they must share some symbol in their prefixes whose length is just above 10% of the shorter.
- Thus, we can index symbols in just the first $\lfloor JL+1 \rfloor$ positions of a string of length L.

Why the Limit on Prefixes?



Extreme case: second string has none of the first E symbols of the first string, but they agree thereafter.

If two strings do not share any of the first E symbols, then $J \geq E/L$.

Thus, $E = JL$ is possible, but any larger E is impossible. Index E+1 positions.

Indexing Prefixes

- Think of a bucket for each possible symbol.
- Each string of length L is placed in the bucket for each of its first $\lfloor JL+1 \rfloor$ positions.
- A B-tree with symbol as key leads to pointers to the strings.

Lookup

- Given a *probe* string *s* of length L, with J the limit on Jaccard distance:

```
for (each symbol a among the
 first ⌊JL+1⌋ positions of s)
    look for other strings in
     the bucket for a;
```

Example: Indexing Prefixes

- Let J = 0.2.
- String abcdef is indexed under *a* and *b*.
- String acdfg is indexed under *a* and *c*.
- String bcde is indexed only under *b*.
- If we search for strings similar to cdef, we need look only in the bucket for *c*.

Using Positions Within Prefixes

- If position i of string s is the first position to match a prefix position of string t, and it matches position j, then the edit distance between s and t is at least $i + j - 2$.
- The LCS of s and t is no longer than L-i+1, where L is the length of s.

Positions/Prefixes – (2)

- If J is the limit on Jaccard distance, then remember $E/(E+C) \leq J$.
  - $E = i + j - 2$.
  - $C = L - i + 1$.
- Thus, $(i + j - 2)/(L + j - 1) \leq J$.
- Or, $j \leq (JL - J - i + 2)/(1 - J)$.

Positions/Prefixes – (3)

- We only need to find a candidate once, so we may as well:
  1. Visit positions of our given string in numerical order, and
  2. Assume that there have been no matches for earlier positions.

Positions/Prefixes – Indexing

- Create a 2-attribute index on (symbol, position).
- If string *s* has symbol *a* as the *i* th position of its prefix, add *s* to the bucket (*a*, *i*).
- A B-tree index with keys ordered first by symbol, then position is excellent.

Lookup

- If we want to find matches for probe string *s* of length L, do:

```
for (i=1; i<=J*L+1; i++) {
  let s have a in position i;
  for (j=1;
     j<=(J*L-J-i+2)/(1-J); j++)
     compare s with strings in
     bucket (a, j);
}
```

Example: Lookup

- Suppose J = 0.2.
- Given probe string adegjkmprz, L=10 and the prefix is ade.
- For the *i* th position of the prefix, we must look at buckets where $j \leq (JL - J - i + 2)/(1 - J) = (3.8 - i)/0.8$.
- For i = 1, $j \leq 3$; for i = 2, $j \leq 2$, and for i = 3, $j \leq 1$.

Example: Lookup – (2)

- Thus, for probe adegjkmprz we look in the following buckets: (*a*, 1), (*a*, 2), (*a*, 3), (*d*, 1), (*d*, 2), (*e*, 1).
- Suppose string *t* is in (*d*, 3). Either:
  - We saw *t*, because *a* is in position 1 or 2, or
  - The edit distance is at least 3 and the length of the LCS is at most 9 (thus the Jaccard distance is at least ¼).

We Win Two Ways

1. Triangular nested loops let us look at only half the possible buckets.
2. Strings that are much longer than the probe string but whose prefixes have a symbol far from the beginning that also appears in the prefix of the probe string are not considered at all.

Adding Length to the Mix

- We can index on three attributes:
  1. Character at a prefix position.
  2. Number of that position.
  3. Length of the *suffix* = number of positions in the entire string to the right of the given position.

Edit Distance

- Suppose we are given probe string *s*, and we find string *t* because its j^{th} position matches the i^{th} position of *s*.
- A lower bound on edit distance E is:

  1. $i + j - 2$ plus
  2. The absolute difference of the lengths of the *suffixes* of *s* and *t* (what follows positions *i* and *j*, respectively).

Longest Common Subsequence

- Suppose we are given probe string *s*, and we find string *t* first because its j^{th} position matches the i^{th} position of *s*.
- If the suffixes of *s* and *t* have lengths *k* and *m*, respectively, then an upper bound on the length C of the LCS is $1 + \min(k, m)$.

Bound on Jaccard Distance

- If J is the limit on Jaccard distance, then $E/(E+C) \leq J$ becomes:
- $i + j - 2 + |k - m| \leq$ $J(i + j - 2 + |k - m| + 1 + \min(k, m))$.
- Thus: $j + |k - m| \leq$ $\big(J(i - 1 + \min(k, m)) - i + 2\big)/(1 - J)$.

Positions/Prefixes/Suffixes – Indexing

- Create a 3-attribute index on (symbol, position, suffix-length).
- If string *s* has symbol *a* as the i^{th} position of its prefix, and the length of the suffix relative to that position is *k*, add *s* to the bucket (*a*, *i* , *k*).

Example: Indexing

- Consider string abcde with J = 0.2.
- Prefix length = 2.
- Index in: (*a*, 1, 4) and (*b*, 2, 3).

Lookup

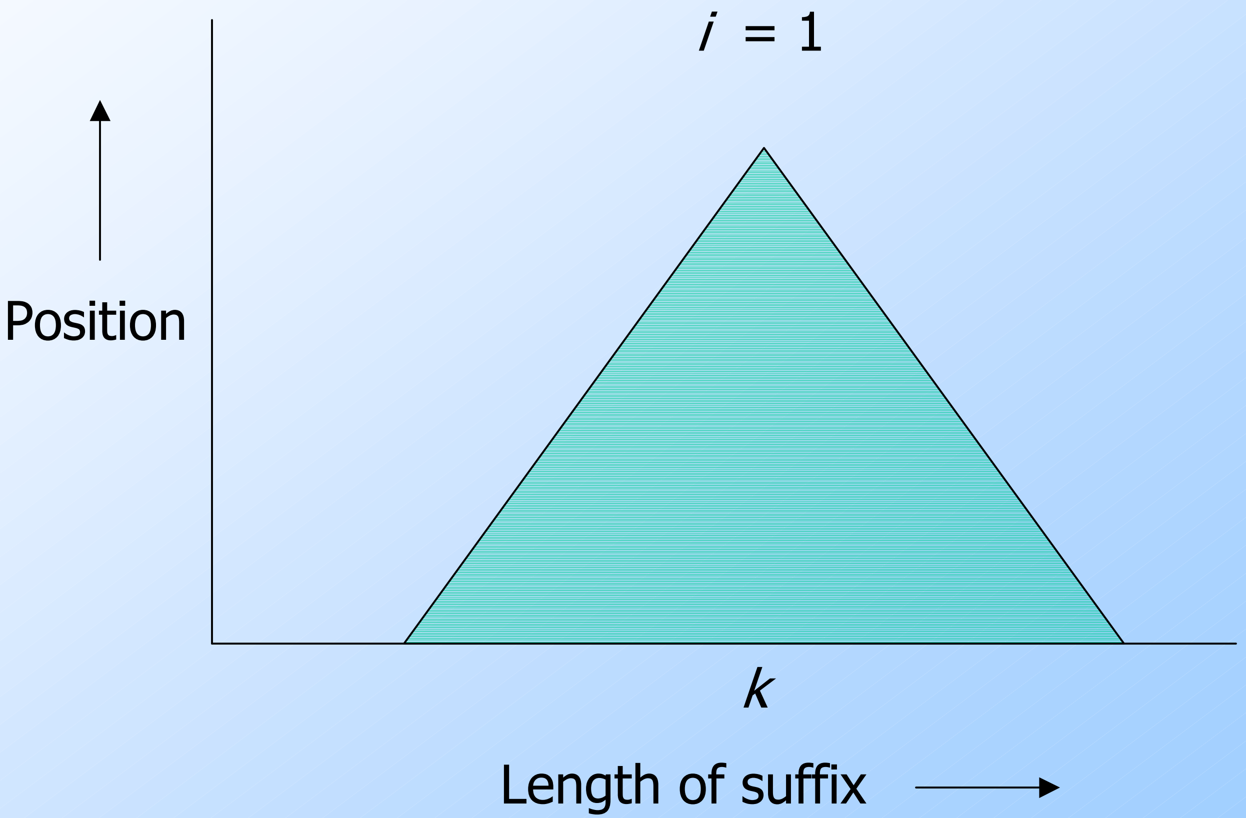
- As for the previous case, to find candidate matches for a probe string *s* of length L, with required similarity J, visit the positions of *s*'s prefix in order.
- If position *i* has symbol *a* and suffix length *k*, look in index bucket (*a*, *j*, *m*) for all *j* and *m* such that $j + |k - m| \leq (J(i - 1 + \min(k, m)) - i + 2)/(1 - J)$.

Example: Lookup

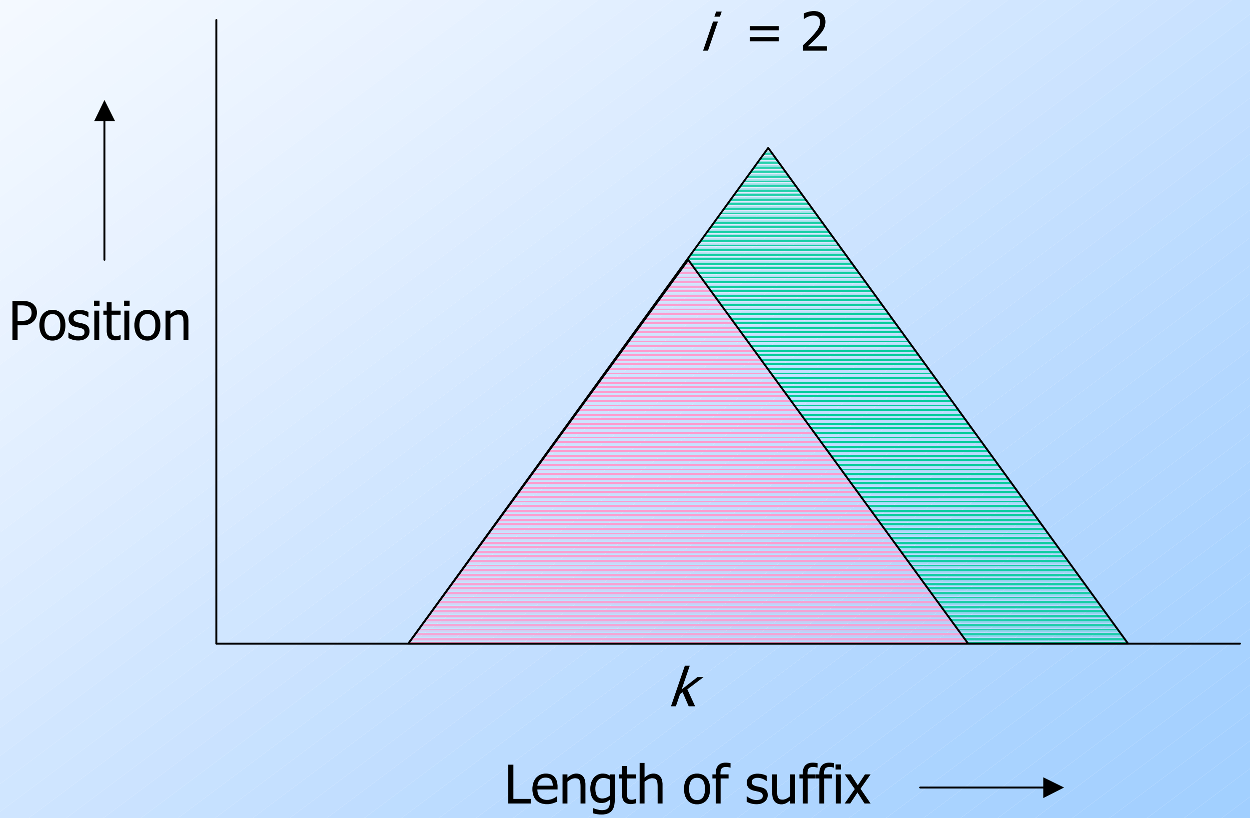
- Consider abcde with J = 0.2.
- Require: $j + |k - m| \leq (J(i - 1 + \min(k, m)) - i + 2)/(1 - J)$.
- For $i = 1$, note $k = 4$. We want $j + |4 - m| \leq (0.2\min(4, m)+1)/0.8$.
- Look in (a, 1, 3), (a, 1, 4), (a, 1, 5), (a, 2, 4), (b, 1, 3).

From $i = 2$, $k = 3$,
$j + |3 - m| \leq 0.2(1+\min(4, m))/0.8$.

Pattern of Search



Pattern of Search



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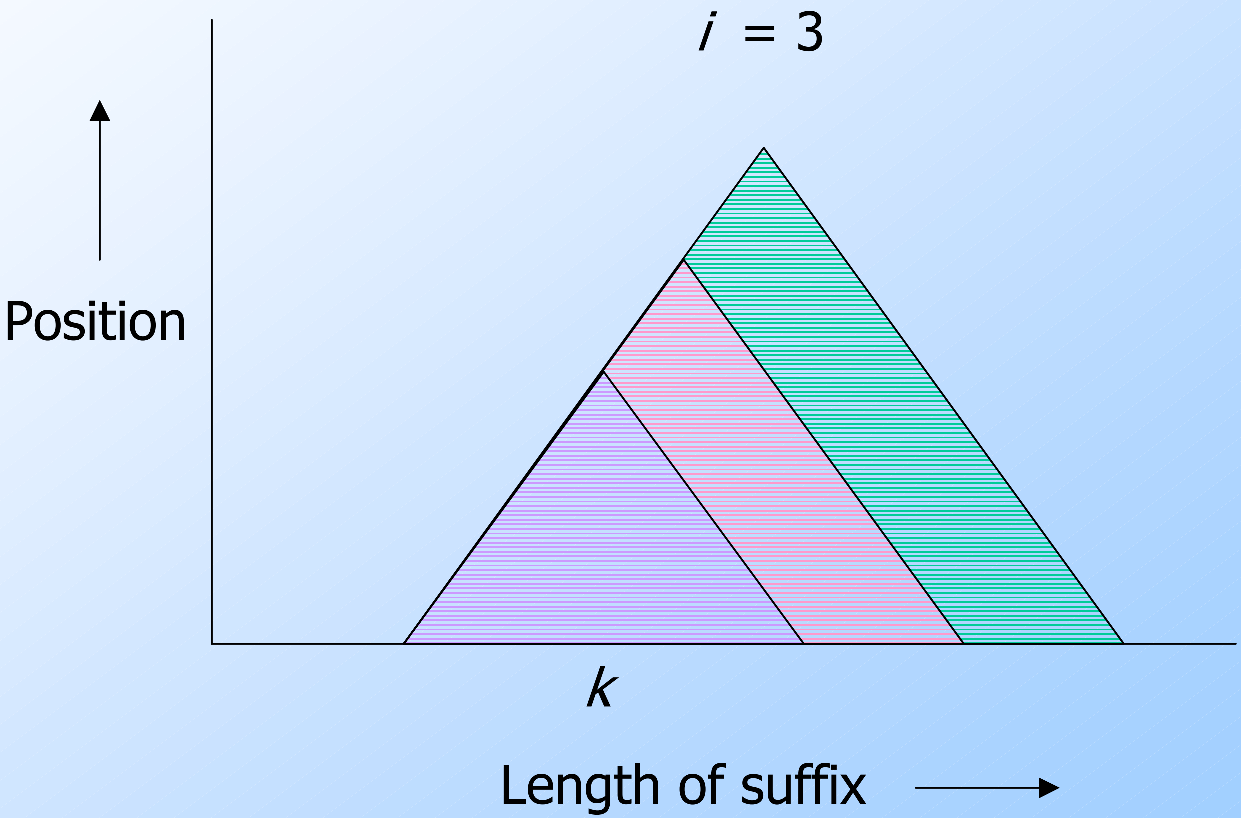

Physical-Index Issues

- A B-tree on (symbol, position, length) isn't perfect.
  - For a given symbol and position, you only want some of the suffix lengths.
  - Similar problem for any order of the attributes.
- Several two-dimensional index structures might work better.