CS345A PROBLEM SET #5

Due in class, Wednesday, May 11, 1994

1) (10 points) Consider the conjunctive queries

Q_1: `p(A,B) :- q(A,C) & r(C,B).`
Q_2: `p(A,B) :- q(B,D) & r(D,A).`

Does the symbol mapping $\tau(B) = A$, $\tau(D) = C$, and $\tau(A) = B$ demonstrate that $Q_1 \subseteq Q_2$? Why or why not?

2) (30 points) Find all equivalences and containments among the four conjunctive queries below. Note that c is a constant in Q_4.

Q_1: `p(X,Y) :- arc(X,A) & arc(A,B) & arc(B,Y)`
Q_2: `p(X,Y) :- arc(X,A) & arc(A,B) & arc(B,C) & arc(C,Y)`
Q_3: `p(X,Y) :- arc(X,A) & arc(B,C) & arc(D,Y) &`
`arc(X,B) & arc(A,C) & arc(C,Y)`
Q_4: `p(X,Y) :- arc(X,A) & arc(A,c) & arc(c,B) & arc(B,Y)`

3) (20 points) Consider the infinite union of conjunctive queries $Q_1 \cup Q_2 \cup \cdots$, where Q_k is

$$\texttt{p(X) :- arc(X,}\mathtt{Z}_1\texttt{) \& arc(}\mathtt{Z}_1\texttt{,}\mathtt{Z}_2\texttt{) \& ... \& arc(}\mathtt{Z}_{k-1}\mathtt{Z}_k\texttt{) \& arc(}\mathtt{Z}_k\texttt{,X)}$$

Note that Q_k represents (not necessarily simple) cycles of length $k+1$ in a directed graph. Find all containments among the Q's.

4) (20 points) Determine whether each of the conjunctive queries

```
p(X,Y) :- arc(X,A) & arc(A,Y) & arc(X,B) &
                arc(B,Y) & X<A & B<Y.
p(X,Y) :- arc(X,Z) & arc(Z,Y) & X<Z & Z<Y.
```

is contained in the other. If not, give a counterexample. *Hint*: The Gupta test is messy here, since there are a lot of containment mappings after we replace duplicate occurrences of variables. However, in general, and here in particular, when there is implication of the OR over all containment mappings, we can usually identify one or a few of the containment mappings that are really involved in the implication. Try first to understand what the CQ's are saying, and then focus on the containment mappings you really need.

5) (20 points) Consider the logic program $\mathcal{P}$ consisting of the rules

```
p(X,Y,Z) :- q(X,Y,Z).
p(X,Y,Z) :- p(A,A,A) & p(A,X,Y) &
                     s(X) & s(Y) & s(Z) & s(A).
```

and the conjunctive query

```
Q: p(X,Y,Z) :- q(B,B,B) & q(B,A,A) & q(A,X,Y) &
                 s(X) & s(Y) & s(Z) & s(A) & s(B).
```

Show that $Q \subseteq \mathcal{P}$.