CS345A Midterm Examination
Monday, May 16, 1994, 2:15 – 3:30 pm

Directions

The exam is *open book*; any written materials may be used. Answer all 6 questions on the exam paper itself. The total number of points is 150. You may make additional comments at the bottoms of pages or at the end if you feel the need to clarify or justify your answers. Do not forget to **sign the pledge** below.

I acknowledge and accept the honor code.

_____

Print your name here: _____

1	2	3
4	5	6

Problem 1: (40 points) Consider the following rules:

```
p(X) :- int(X) & X >= 2 & not c(X)
c(X) :- int(X) & p(Y) & divides(X,Y) & X != Y
```

Think of $p(X)$ as meaning "X is a prime" and $c(X)$ as "X is composite." The EDB predicate $int(X)$ says that X is a positive integer, and in practice it will hold a finite set of integers. The EDB predicate $divides(X,Y)$ means that Y evenly divides X.

Suppose that $int = \{1,2,3,4\}$, and *divides* is the expected relation on these four integers; that is, $divides = \{(1,1), (2,1), (3,1), (4,1), (2,2), (4,2), (3,3), (4,4)\}$. If we instantiate these rules in all possible ways, eliminate rules with a known false subgoal and then eliminate known true subgoals from the remaining rules, we are left with the following:

```
p(2) :- not c(2)     c(2) :- p(1)
p(3) :- not c(3)     c(3) :- p(1)
p(4) :- not c(4)     c(4) :- p(1)
                     c(4) :- p(2)
```

a) In the table at the top of the next page, construct the well-founded model by the definition (not the alternating fixedpoint). Indicate what inferences are made about the truth or falsehood of $p(i)$ and $c(i)$, for $1 \leq i \leq 4$ at each round.

Round	Positive Inferences	Negative Inferences
1		
2		
3		
4		

b) In the space below, draw the dependency graph for the instantiated atoms $p(i)$ and $c(i)$ for $1 \leq i \leq 4$.

c) Are the rules with the given EDB locally stratified? _____ If so, tell what the strata are; if not, describe an infinite negative path.

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d) Suppose *int* contains the integers from 1 to n, and *divides* contains all those pairs (i, j) such that j divides i and i and j are integers between 1 and n. For what values of n will the rules and EDB be locally stratified? Explain briefly.

Problem 2: (30 points) Consider the following conjunctive queries:

Q_1: `h(X) :- red(X,Y) & blue(Y,Y) & red(Y,Z)`
Q_2: `h(X) :- red(X,Y) & blue(Y,Z) & red(Z,W)`
Q_3: `h(X) :- red(X,Y) & red(Y,Z)`

In the table below, for each containment, either give a containment mapping that proves it or give a database that serves as a counterexample.

Containment	Counterexample or Containment Mapping
$Q_1 \subseteq Q_2$	
$Q_1 \subseteq Q_3$	
$Q_2 \subseteq Q_1$	
$Q_2 \subseteq Q_3$	
$Q_3 \subseteq Q_1$	
$Q_3 \subseteq Q_2$	

Problem 3: (20 points) Below are two conjunctive queries with arithmetic comparisons.

Q_1': **panic** `:- arc(X,Y) & arc(Y,Z) & X <= Y & Y <= Z`
Q_2': **panic** `:- arc(X,Y) & arc(Y,Z) & X <= Z`

To prove that $Q_1' \subseteq Q_2'$ we begin by splitting the double use of Y in each of the conjunctive queries:

Q_1: **panic** `:- arc(X,Y) & arc(W,Z) & W=Y & X <= Y & Y <= Z`
Q_2: **panic** `:- arc(X,Y) & arc(W,Z) & W=Y & X <= Z`

a) What are all the containment mappings from the ordinary subgoals of Q_2 to the ordinary subgoals of Q_1? Name them and list them here:

b) The "Gupta test" for $Q_1 \subseteq Q_2$ requires that we write down a logical expression. Write that expression here:

c) Briefly explain why your expression from (b) holds. *Hint*: The implication of disjunction (logical OR) can be shown true by showing that the hypothesis implies any *one* of the disjuncts.

Problem 4: (20 points) For the rules

```
p(X,Y,Z) :- a(X,Y,Z)
p(X,Y,Z) :- p(X,Y,W) & p(Y,W,Z)
```

draw the rule goal graph starting with the goal p^{bff} in the space below.

Problem 5: (20 points) Convert the following rules:

```
p(X,Y) :- a(X,Y)
p(X,Y) :- p(X,Z) & p(Z,Z) & p(Z,Y)
```

into equivalent rules with rectified subgoals.

Problem 6: (20 points) For the query $p(0, Z)$, apply a magic-sets transformation to the following rules:

r_1: `p(X,Y) :- c(X,Y)`
r_2: `p(X,Y) :- a(X,U) & p(U,V) & b(V,W) & p(W,Y)`

You may use either the basic transformation or a simplified transformation that eliminates some supplementary predicates.