Rule/Goal Trees

Nodes correspond to rules and to subgoals of rules.

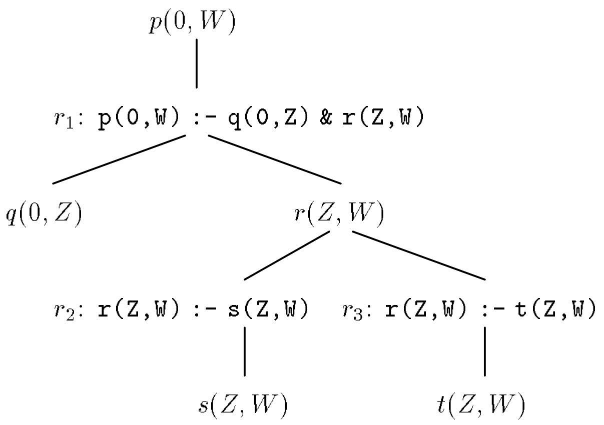
- Rule node: children = subgoals of the rule.
- Goal node: children = rules whose heads unify with the goal. Unifying substitution must be made in the rule.
    - ❖ Be careful that local variables of the rule are changed so their are no accidental equalities between local variables and variables of the parent goal.
- Root is goal node corresponding to a query. Arguments of goal may have constants acquired from query.

Example

Page 769 of PDKS-II has suggestion of infinite rule/goal tree from recursive example. Here is a simple, nonrecursive example.

r_1: `p(X,Y) :- q(X,Z) & r(Z,Y)`
r_2: `r(A,B) :- s(A,B)`
r_3: `r(A,B) :- t(A,B)`

Query: $p(0,W)$? i.e., for what values of W is $p(0,W)$ true?



Passing Relations Around the Rule/Goal Tree

We evaluate a query by passing information around the R/G tree in three ways.

- Queries are passed down. A query gives bindings for certain arguments of a goal.
- Answers are passed upward. These relations give values for the arguments of a goal.

- Information is passed sideways, from left to right, through the subgoals of a rule.

Supplementary Relations

Sideways information passing is accomplished through "supplementary relations."

- If a rule has n subgoals, there are n supplementary relations, $S_0, S_1, \ldots, S_{n-1}$.
- The supplementary relation S_i has arguments corresponding to certain variables of the rule. These variables must:
  1. Have been bound before the $(i+1)$st subgoal, by appearing either in a bound argument of the head or in the ith or earlier (to the left) subgoal.
  2. Be used later; either the variable appears in the $(i+1)$st or later (to the right) subgoal, or anywhere in the head.

Example

In our R/G tree, the rule node labeled r_1 has supplementary relations S_0 and S_1.

- S_0 has no arguments, because although the first argument of the head is bound by the query, there are no variables bound.
- S_1 has only Z as an argument, because Z appears in the first subgoal, $q(0, Z)$, and is used later, in the second subgoal $r(Z, W)$.

The nodes labeled r_2 and r_3 each have only S_0 as a supplementary relation.

- Its argument is Z, because Z is bound by the head (as we shall see) and used in the first and only subgoal.

Bound Arguments and Variables

- Important: Goals have bound arguments; rules have bound variables.
- Distinction is important in many situations, e.g., function symbols, constant arguments, duplicate occurrences of variables.

ATOV

- Match is tuple-by-tuple.
- Match of argument against a term (an argument of a rule head) must be exact; i.e., argument and term must unify.

- If a variable is bound to two different constant values, match fails.

Example

Rule head `p(f(X,Y),Z,W) :-`. First two arguments are bound by the relation

Arg #1	Arg #2
$f(a,b)$	c
d	$g(e)$

- First tuple: $X \rightarrow a$, $Y \rightarrow b$, $Z \rightarrow c$.
- Second tuple: no match. $f(X,Y)$ does not unify with d.
- Result:

X	Y	Z
a	b	c

Example

Head `p(X,X,Y,Z) :-`, with first three arguments bound by the relation

Arg #1	Arg #2	Arg #3
a	b	c
a	a	$g(d)$
b	b	c

- First tuple: no match; X cannot be both a and b.
- Second tuple: $X \rightarrow a$, $Y \rightarrow g(d)$.
- Third tuple: $X \rightarrow b$, $Y \rightarrow c$.
- Result:

X	Y
a	$g(d)$
b	c

VTOA

- An argument must become completely bound, or it is not bound at all.

Example

Subgoal `p(f(X,Y),Z,W)` with X, Y, and Z bound by:

X	Y	Z
a	b	c
d	e	$f(g)$

- Result:

Arg #1	Arg #2
$f(a,b)$	c
$f(d,e)$	$f(g)$

Example

Subgoal `p(f(X,Z),X,Y)` with X and Y bound.

- The first argument is *not* bound, because Z is not bound. The second and third arguments are bound.
- Variable-binding and result relations:

X	Y
a	b
c	d

Arg #2	Arg #3
a	b
c	d

Rule/Goal Graphs

- Condense nodes of rule/goal tree having same rule or goal with the same binding pattern.

Goal Nodes

- Predicate + "adornment."
- *Adornment* = list of b's and f's, indicating which arguments are bound, which are free.
- Example: p^{bfb}. First and third arguments of p are bound.

Rule Nodes

- Correspond to supplementary relation.
- $r_i^{[S|T]}$ represents the point in rule r after seeing i subgoals, with variables in set S bound, those in T free.

Children

Children of goal node p^α are those rule nodes $r_0^{[S|T]}$ such that

1. Rule r has head predicate p.

2. S is the set of variables that appear in those arguments of the head that α says are bound.

3. T is the other variables of r.

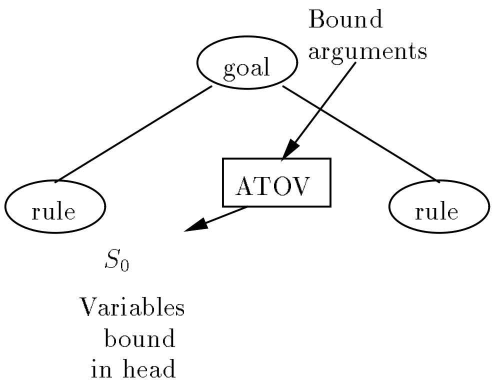
Children of the rule node $r_j^{[S|T]}$ are:

1. The goal node of the $(j+1)$st subgoal of r, with adornment that binds those arguments whose only variables are in S.

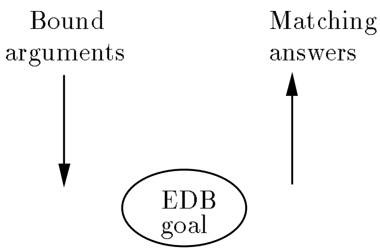
2. The rule node $r_{j+1}^{[S'|T']}$, where $S' = S +$ variables appearing the in $(j+1)$st subgoal; T' is the other variables.

- Exceptions: no r_{j+1} rule node if r has only $j+1$ subgoals. No goal child if $j = 0$ and r has no subgoals.

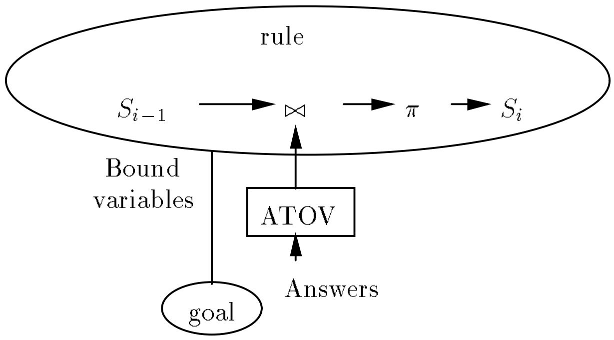
Widgets



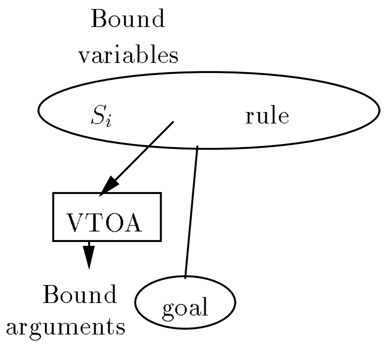
Passing queries from a goal to its rule children



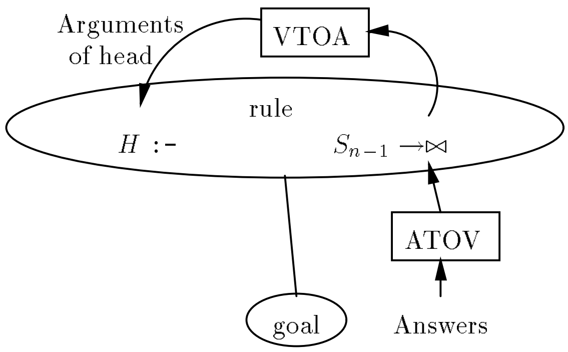
Interrogating the database



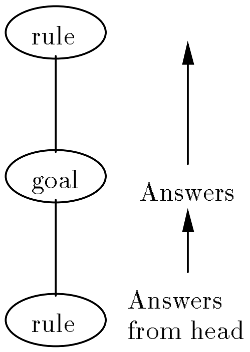
Joining the result of a goal into the supplementary relations



Querying the next subgoal



Answers for the last goal are joined with the last supplementary relation and produce tuples for the head



Answers from the head of a rule are passed to the goal parent and from there to the rule parent of that goal