CS109A Quiz

Monday, Nov. 26, 10–10:50 am

Directions

The exam is *open book*; any written materials may be used. Answer all 6 questions on the exam paper itself. The total number of points is 100. You may make additional comments at the bottoms of pages or at the end if you feel the need to clarify or justify your answers. Do not forget to **sign the pledge** below.

I acknowledge and accept the honor code.

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Print your name here: ____

Problem 1: (20 points) Suppose we represent sets by sorted lists of elements. An element may appear only once on a list. Write an ML function `union(S,T)` that computes the union of two sets S and T represented in this way and returns the result as a sorted list with no duplicates.

Problem 2: (20 points) A node of a binary tree is a leaf if it has neither left nor right children. A node of a binary tree is said to be *full* if it has both left and right children. Prove by structural induction that in any binary tree of one or more nodes, there is one more leaf than full nodes. Your proof should not mention the number of nodes in the tree explicitly, except to distinguish between leaves and interior nodes.

a) What is the basis case?

b) Prove the basis case here.

c) State the inductive step here.

d) Prove the inductive step here.

Problem 3: (20 points) Suppose we use the usual type declarations for cells and lists,

```
type CELL = record
        element: ETYPE;
        next: ↑ CELL
    end;
    LIST = ↑ CELL
```

Consider the sketch of a procedure `foo` below.

```
procedure foo(L: LIST);
begin
    if L = NIL then something O(1)
    else begin
        something O(n^2);
        foo(L↑.next);
        something O(n^3)
    end
end
```

a) Write a recurrence relation for $T(n)$, the time taken by `foo` when its argument L is a list of length n.

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b) Give a big-oh solution to your answer to (a). You may look the answer up in the notes; you do not have to expand the recurrence.

Problem 4: (10 points) In the space below, draw a labeled binary tree with the following properties.

i. The tree has four nodes.
ii. The labels satisfy the binary-search-tree property.
iii. The labels satisfy the partially-ordered tree property.

Problem 5: (15 points) If $S \subseteq T$ for sets S and T, then $S \cup T = T$. Prove this fact by showing, under the assumption that $S \subseteq T$:

a) If x is in T, then x is in $S \cup T$.

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b) If x is in $S \cup T$, then x is in T.

Problem 6: (15 points) For the infix expression $((x + y) * (x + z)) * (y - z)$

a) Draw an expression tree in the space below.

b) Convert the infix expression to a prefix expression.

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c) Convert the infix expression to a postfix expression.
