CS109B Quiz

Monday, Feb. 11, 10–10:50 am

Directions

The exam is *open book*; any written materials may be used. Answer all 6 questions on the exam paper itself. The total number of points is 100. You may make additional comments at the bottoms of pages or at the end if you feel the need to clarify or justify your answers. Do not forget to **sign the pledge** below.

I acknowledge and accept the honor code.

____

Print your name here: ____

1	/20
2	/15
3	/15
4	/20
5	/15
6	/15
Total	/100

Problem 1: (20 points) Write a grammar for the language consisting of nonempty strings of alternating `a`'s and `b`'s. Examples of such strings are `a`, `b`, `ab`, `ba`, `aba`, `bab`, and so on.

Explain what the language of each of your syntactic categories is.

Problem 2: (15 points) Consider the grammar

$$< S > \rightarrow \mathbf{a} < S > \mid < S > \mathbf{b} \mid \epsilon$$

a) Give a regular expression whose language is $L(< S >)$.

---

b) How many parse trees are there in this grammar for the string `aab`?________________

c) Draw them each below.

Problem 3: (15 points) Here is another grammar; it happens to generate the language of the regular expression $\mathbf{a}^*\mathbf{b}^*\mathbf{c}$.

(1) $<S> \rightarrow <T><R>\ \mathbf{c}$
(2) $<T> \rightarrow \mathbf{a} <T>$
(3) $<T> \rightarrow\ \epsilon$
(4) $<R> \rightarrow \mathbf{b} <R>$
(5) $<R> \rightarrow\ \epsilon$

This grammar can be parsed by a table-driven parser of the type described in Section 11.7.

a) How does one decide which production for $< S >$ to use?

---

b) How does one decide which production for $< T >$ to use?

---

c) How does one decide which production for $< R >$ to use?

---

d) Draw the parsing table in the space below.

Problem 4: (20 points) Below is a Karnaugh map.

$pq \backslash rs$	00	01	11	10
00	1	1	0	1
01	1	0	0	1
11	0	1	1	0
10	0	1	1	0

a) Circle all the prime implicants.

b) List the prime implicants, giving the AND-of-literals form for each.

---

c) Only a subset of the prime implicants are needed to cover the map, that is, to include all the points with 1's. Give a minimal such set below.

Problem 5: (15 points) Let E be the logical expression

$$\Big((p \rightarrow x) \ \texttt{AND}\ \big(p \rightarrow (x \rightarrow y)\big)\Big) \rightarrow (p \rightarrow y)$$

a) In the space below, draw the expression tree for E (called "syntax tree" in the notes).

b) Construct the truth table for all the subexpressions of E, including E itself. Is E a tautology?______________ You may construct the various columns of the truth table in the space provided below. You should number the nodes of your tree from (a) and use these numbers to head the columns of the truth table.

p	x	y	
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Problem 6: (15 points) Let F be the expression

$\texttt{NOT}\big((x \texttt{ AND } y) \texttt{ OR } (x \texttt{ AND } z)\big)$

a) In the space below, show how to use DeMorgan's laws to transform F into an expression in which the only uses of `NOT` apply to single variables (not to larger expressions). Each application of the law should be shown as a single step.

b) Use one or more of the algebraic laws from Section 12.8 to transform your answer from (a) to a simpler form. In particular, your final expression should have no more than five operator occurrences.