CS109A Class Quiz

Friday, Feb. 3, 1995, 3:15–4:05PM

Directions

The exam is *open book*; any written materials may be used. Answer all 7 questions on the exam paper itself. The total number of points is 100. You may make additional comments at the bottoms of pages or at the end if you feel the need to clarify or justify your answers. Do not forget to **sign the pledge** below.

I acknowledge and accept the honor code.

____

Print your name here: ____

1	2	3	4
5	6	7	

Problem 1: (20 points) We wish to prove the statement

$S(n)$: $n! < n^n$

The statement is meaningless for $n = 0$ and false for $n = 1$. However, for other n, we can prove it by induction on n. To help organize your proof, answer the following:

a) What is a suitable basis case? ______________

b) Prove the basis case.

c) When we prove the inductive step, we assume $S(n)$ and must prove $S(n+1)$. What is $S(n+1)$?

d) Prove the inductive step. *Note*: Remember to make your proof "go the right way." Start with things known true and derive what you want to prove.

Problem 2: (15 points) Below is an ML function `longer` that tests whether its first argument is a strictly longer list than its second argument.

```
(1)    fun longer(nil,L) = false
(2)    |   longer(L,nil) = true
(3)    |   longer(x::xs, y::ys) = longer(xs,ys);
```

We wish to prove that the value of $longer(L_1, L_2)$ is `true` if and only if L_1 is strictly longer than L_2. The proof is an induction on the length n of L_1.

a) What is the statement $S(n)$ to be proved?

b) What is a suitable basis case? ______________

c) Prove the basis case.

d) Prove the inductive step (on the next page). *Note*: Note that sometimes, when proving the inductive step, we encounter a situation at line 2 where `longer` does not make a recursive call. There is nothing wrong here; the inductive part of the proof and the recursive part of the program do not have to match exactly.

Problem 3: (5 points) What is the type of the function `longer` from Problem 2? *Note*: You may use any type variables (identifiers beginning with apostrophe) in your answer; you do not have to guess the type variables SML/NJ would choose.

Problem 4: (20 points) Write a function that flips alternate elements of a list. That is, given a list $[a_1, a_2, \ldots, a_n]$ as argument, produce $[a_2, a_1, a_4, a_3, a_6, a_5, \ldots]$. If n is odd, a_n remains at the end. *Hint*: This function is easy if you use the right patterns in its rules.

Problem 5: (10 points) There are many pairs of witnesses (n_0, c) to the fact that $n^2 + 3n + 5$ is $O(n^2)$.

a) What is the minimum value of c such that $(2, c)$ is a witness pair? ____________

b) What is the minimum value of n_0 such that $(n_0, 2)$ is a witness pair? ____________

Problem 6: (15 points) Below is a C function `weird` that takes two arguments a and b; you may assume it will only be called with $a \leq b$. It prints a zero if $a = b$ and otherwise calls itself twice, with arguments $(a+1, b)$ and $(a, b-1)$.

```
    void weird(int a, int b) {
(1)     if(a==b)
(2)         putchar('0');
(3)     else {
(4)         weird(a+1,b);
(5)         weird(a,b-1);
        }
    }
```

a) In the space below, draw the structure tree for this function. Indicate the (simple and complex) statements and the line numbers they represent. (*Note*: a function call by itself may be considered a simple statement.) Assuming that the input size measure n is $n = b - a$ and that $T(n)$ is the running time of a call to `weird`, give tight bounds on the running time of each node in your structure tree.

b) Write a recurrence relation for the running time $T(n)$ of `weird`. Do *not* solve the recurrence.

Problem 7: (15 points) Consider the recurrence

Basis: $T(0) = 2$.

Induction: $T(n) = nT(n-1)$ for integers $n > 0$.

Note that since there are no symbolic constants, this is not a "big-oh" problem; we can solve for $T(n)$ exactly.

a) To begin, what are the initial values of $T(n)$?

$T(0) =$ ______; $T(1) =$ ______; $T(2) =$ ______; $T(3) =$ ______; $T(4) =$ ______.

b) Expand the inductive rule so $T(n)$ is expressed in terms of $T(n-2)$.

____

c) Express $T(n)$ in terms of $T(n-3)$.

____

d) What is the general pattern? That is, express $T(n)$ in terms of $T(n-i)$.

____

e) For what value of i can we eliminate $T(n-i)$ from the expression? $i =$ ______

f) Use your answer to (e) to express $T(n)$ as a function of n alone (i.e., without any terms involving the function T).

$T(n) =$ ____________.