CS109B Class Quiz

Monday, May 1, 1995, 2:15–3:05PM

Directions

The exam is *open book*; any written materials may be used. Answer all 7 questions on the exam paper itself. The total number of points is 100. You may make additional comments at the bottoms of pages or at the end if you feel the need to clarify or justify your answers.

Do not forget to **sign the pledge** below.

I acknowledge and accept the honor code.

____

Print your name here: ____

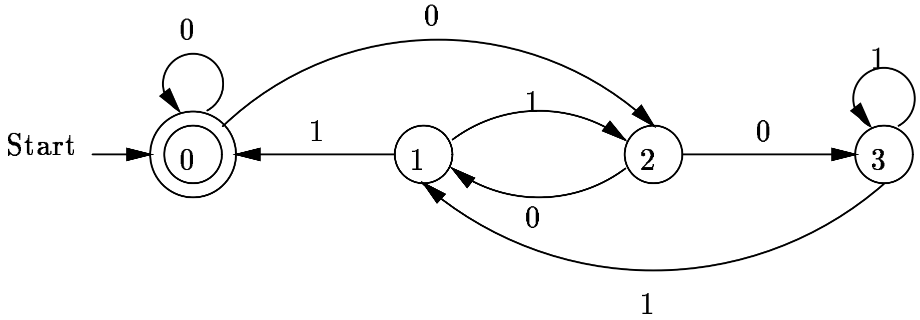
1	2	3	4
5	6	7	

Problem 1: (15 points) Consider an undirected graph with 38 nodes named $1, 2, 3, \ldots, 30$, 31, *Sunday, Monday, Tuesday, Wednesday, Thursday, Friday*, and *Saturday*. An edge connects a number-node n and a day-node d if May n, 1995 is that day of the week d. For example, there is an edge between nodes 1 and *Monday*.

a) How many connected components does this graph have? ________ How many nodes are in the largest connected component? ________

b) Is this graph planar? ________ Justify your answer below.

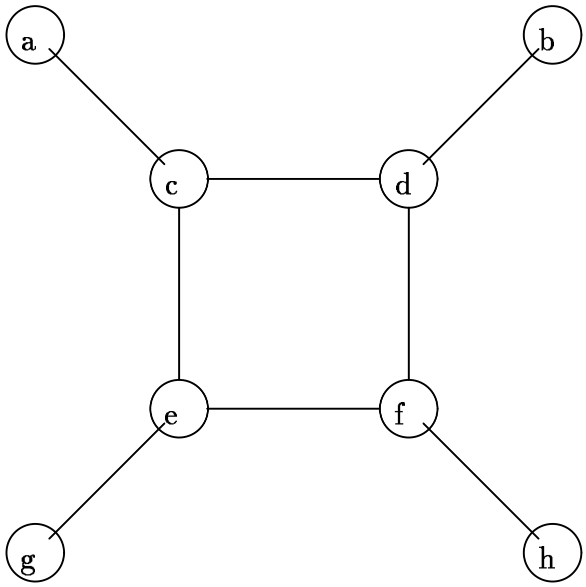
Problem 2: (15 points) Below is a nondeterministic automaton.



In the space below, apply the subset construction to this automaton to produce an equivalent deterministic automaton.

Problem 3: (15 points) In the space below, draw two different depth-first spanning trees for the graph of Problem 2, each starting at node 0. (Ignore arc-labels, of course.) Indicate on each tree those arcs that are forward, backward, and cross; i.e., write "F" next to an arc that is a forward arc, "B" for a backward arc, and "C" for a cross arc. (Write no letter on the tree arcs.)

Problem 4: (20 points) Consider the following undirected graph.



Answer the following questions about this graph:

a) Is the graph bipartite? ________ Tripartite? ________

b) What is the largest number of nodes in an independent set? ________ Below list one such maximal independent set.

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c) What is the fewest nodes in a node cover? ________ Below list a minimal node cover.

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d) Assuming all edges have label (weight) 1, how many different minimal-weight spanning trees are there? ________ Optionally, justify your answer for partial credit.

Problem 5: (15 points) ML quickies:

a) Write a statement defining `recType` to be a type that is a record with a field `node` that is an integer and a field `adj` that is a list of integers.

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b) Complete the case-expression below so that its value is `true` if x is a nonempty string (not list) and `false` if x is the empty string.

```
case x of
```

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c) On the two lines below, give the ML responses. You need not give responses to statements not followed by a horizontal line.

```
val num = ref 0;

num := 1;

!num;
```

```
num;
```

Problem 6: (15 points) Write regular expressions for the following languages. *Optional Shorthand*: You may use X for the regular expression $(a \mid b)^*$.

a) The set of all strings of a's and b's containing the **substring** abb.

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b) The set of all strings of a's and b's of which a, b, b is a **subsequence**.

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c) The set of all strings of a's and b's containing at least one a and two b's (in any order).

Problem 7: (15 points) Prove that $L(R \mid S) \subseteq L(R^* S^*)$. Note that this set inclusion is not an equivalence; the inclusion in the other direction is false. *Hint*: Begin by assuming that w is a string in $L(R \mid S)$. (You may start your proof assuming that was already said.)