CS109A Class Quiz

Friday, Feb. 2, 1996, 3:15–4:05PM

Directions

The exam is *open book*; any written materials may be used. Answer all 7 questions on the exam paper itself. The total number of points is 100. You may make additional comments at the bottoms of pages or at the end if you feel the need to clarify or justify your answers. Do not forget to **sign the pledge** below.

I acknowledge and accept the honor code.

____

Print your name here: ____

1	2	3	4
5	6	7	

Problem 1: (10 points) Convert the logical expression $(p \rightarrow q) \rightarrow r$ to an equivalent expression that uses only the operators OR and NOT (which you may represent in any of the notations used in class).

Now, convert your expression to an equivalent form that uses AND, OR, and NOT, but where the NOT operator(s) apply only to propositional variables, not to any larger subexpression.

Problem 2: (15 points) In each of the following lines, one of the three expressions is not equivalent to the other two. Circle the one that is not equivalent to the others on its line.

a) $p \rightarrow (p \rightarrow q)$ $\qquad$ $p \rightarrow (q \rightarrow p)$ $\qquad$ $q \rightarrow (p \rightarrow q)$

b) $\texttt{NOT}\big((\forall X)p(X)\big)$ $\qquad$ $(\forall X)\big(\texttt{NOT}\ p(X)\big)$ $\qquad$ $(\exists X)\big(\texttt{NOT}\ p(X)\big)$

c) $(p \rightarrow q) \rightarrow (q \rightarrow p)$ $\qquad$ $q \rightarrow p$ $\qquad$ $(\texttt{NOT}\ p) + q$

Problem 3: (15 points) The following ML function `max` computes the maximum integer on a list.

```
fun max([x:int]) = x
|   max(x::xs) =
        let
            val m = max(xs);
        in
            if x>m then x else m
        end;
```

Prove that fact by induction.

a Give the statement $S(n)$ to be proved. Be sure to make clear what n is.

---

b) What is an appropriate basis case? ____________

c) Prove the basis case.

---

d) Prove the inductive step $S(n) \rightarrow S(n+1)$.

Problem 4: (10 points) The function `max` as written in Problem 3 will cause ML to warn us that the "match is nonexhaustive." Explain briefly what the problem is (i.e., what kind of input is not matched) and how we can modify the function to match any input, without ever returning a false answer.

Problem 5: (15 points) The class of *Goofy strings* is defined recursively as follows:

Basis: 0 and 1 are Goofy strings.

Induction:

1. If w is a Goofy string, then so are $0w$ and $w1$. That is, we get another Goofy string when we either attach a 0 to the *beginning* of a Goofy string or a 1 to the *end* of a Goofy string.
2. If w is a Goofy string, then w^R, the reverse of w, is also a Goofy string.

For example, 0 and 1 are Goofy by the basis. By inductive rule (1), 01 is Goofy (prepend 0 to Goofy string 1). By inductive rule (1) again, 011 is Goofy (append 1 to Goofy string 01). By inductive rule (2), 110 is Goofy (reverse Goofy string 011).

We would like to prove the following for all $n \geq 1$:

$S(n)$: Every string of 0's and 1's having length n is a Goofy string.

a) Prove the basis. Do not forget to indicate what value(s) of n form the basis case(s).

b) Prove the inductive step $S(n) \rightarrow S(n+1)$.

Problem 6: (20 points) Write an ML function `samePos(L1,L2)` that takes two lists of integers and returns the list of values that appear in the same positions of the two lists. For example, given the two lists `L1 = [1,3,2,5,3,8,6]` and `L2 = [8,3,7,5,3,6]`, return the list `[3,5,3]`, because the second positions of both lists have 3, the fourth positions of both lists have 5, the fifth positions each have 3, but no other positions of the two lists agree. Note that the two lists do not have to be of the same length.

Problem 7: (15 points) For each of the following pairs of functions, tell whether the first is big-oh of the second. If not, just say "no." If the first *is* big-oh of the second, say "yes" and give **minimal** witnesses n_0 and c. By "minimal," we mean that lowering either n_0 or c while keeping the other fixed results in a pair that is *not* a witness. Note that there are usually many different choices of minimal witnesses when the big-oh relationship holds.

a) Is 3^n $O(2^{2n})$?

b) Is $n + \log_2 n$ $O(n)$?

c) Is $10n^2$ $O(n^2 - n)$?