CS109B Problem Set #2
Due 4:30 PM, Friday 4/21/95

1) (10 points) Consider the graph with three nodes, a, b, and c, and the two arcs $a \rightarrow b$ and $b \rightarrow c$. Give all the possible depth-first search forests for this graph, considering all possible starting nodes for each tree. What is the postorder numbering of the nodes for each forest? Are the postorder numbers always the same for this graph? (Exercise 9.6.5,FCS)

2) (20 points) Consider the generalization of the graph of Exercise 1 to a graph with n nodes, $a_1, a_2, \ldots, a_n$, and arcs $a_1 \rightarrow a_2$, $a_2 \rightarrow a_3, \ldots, a_{n-1} \rightarrow a_n$. Prove by complete induction on n that this graph has 2^{n-1} different depth-first search forests. *Hint*: It helps to remember that $1 + 1 + 2 + 4 + 8 + \cdots + 2^i = 2^{i+1}$, for $i \geq 0$. (Exercise 9.6.6, FCS)

3) (15 points) Suppose we have a directed graph G, from whose representation we have just constructed a depth-first spanning forest F by the algorithm of Fig. 9.31. Let us now add the arc $u \rightarrow v$ to G to form a new graph H, whose representation is exactly that of G, except that node v now appears somewhere on the adjacency list for node u. If we now run Fig. 9.31 on this representation of H, under what circumstances will the same depth-first forest F be constructed? That is, when will the tree arcs for H be exactly the same as the tree arcs for G? Justify your answer. (Exercise 9.6.8, FCS)

4) (10 points) Consider the graph with arcs $a \rightarrow c$, $b \rightarrow a$, $b \rightarrow c$, $d \rightarrow a$, and $e \rightarrow c$. Find all the topological orders for this graph. (Exercise 9.7.4(b), FCS)

5) (20 points) Give an $O(m)$ algorithm to find in a directed, acyclic graph for all nodes u the longest path from anywhere to u. (Exercise 9.7.6(b), FCS) *Hint*: Adapt the idea from the 4/10 class problem.

6) (15 points) Suppose a directed graph, whose nodes are integers from 0 to $n-1$, is represented by adjacency lists, that is, an array with n elements. The ith element is a list of the successors of node i. Write an ML function `addEdge(u,v,A)` that adds v to the list of successors of u. You need not check that v is not already there.

7) (10 points) Use Floyd's algorithm to find the shortest possible time between each of the hominid species in the table on the next page. Show your work, i.e., the changes to the matrix at each pivoting step. (Exercise 9.9.4, based on Fig. 9.48)

Species 1	Species 2	Time
AF	HH	1.0
AF	AA	0.8
HH	HE	1.2
HH	AB	0.5
HH	AR	0.3
AA	AB	0.4
AA	AR	0.6
AB	HS	1.7
HE	HS	0.8