CS109B Problem Set #6
Due 4:30 PM, Friday 6/2/95

Because of the exam and holiday, I have again consolidated work into one large assignment worth 150 points. **Note there is a 9th problem on the reverse side.**

1) (15 points) Use laws given in Section 12.8 to transform $pq + rs$ into

$$(p + r)(p + s)(q + r)(q + s)$$

To save effort, you may omit steps that use laws 12.5 through 12.13, which are analogous to arithmetic. For example, commutativity and associativity of `AND` and `OR` may be assumed. (Exercise 12.8.4(a), FCS)

2) (20 points) We would like to prove the theorem "If x is a perfect square and x is even, then x is divisible by 4."

   a) Designate propositional variables to stand for the three conditions about x mentioned in the theorem.
   b) Write the theorem formally in terms of these propositions.
   c) State the contrapositive of your answer to (b), both in terms of your propositional variables and in colloquial terms.
   d) Prove the statement from (c). *Hint*: It helps to notice that if x is not divisible by 4, then either x is odd, or $x = 2y$ and y is odd. (Exercise 12.9.2, FCS)

3) (15 points) Prove that $p + \bar{q}\bar{r} + \bar{p}r + q\bar{r}$ is a tautology by showing it is equivalent to `TRUE`. (Exercise 12.9.5(a), FCS)

4) (15 points) Deduce from hypotheses p and $p \rightarrow \bar{p}$ the conclusion 0. You may use inference rules (a) through (d) from Section 12.10, and you may use any (substituted instance of) any tautology from Sections 12.8 and 12.9.

5) (15 points) Give a proof by resolution of the fact that $p \rightarrow q$ and $p \rightarrow r$ imply $p \rightarrow qr$. (Exercise 12.11.4(a), FCS)

6) (15 points) Design a circuit for the function $xy + xz + yz$. (Exercise 13.4.1(b), FCS)

7) (15 points) First-string commandos are ready, willing, and able. Suppose we have n commandos, and circuit inputs r_i, w_i, and a_i indicate, respectively, whether the ith commando, is ready, willing, and able. We only want to send the commando team on a raid if they all are ready, willing, and able. Design a divide-and-conquer circuit to indicate whether we can send the team on a raid. (Exercise 13.5.4, FCS)

8) (20 points) Second-string commandos aren't as professional. We are willing to send them on a raid if each is either ready, willing, or able. In fact, we'll send the team even if at most one of the commandos is neither ready, willing, nor able. Using the same inputs as Problem 7, devise a divide-and-conquer circuit that will indicate whether we can send the second-string commando team on a raid.

9) (20 points) Below is the `Stack` structure we had in the section notes for the week of 5/8.

```
structure Stack = struct
    exception EmptyStack;

    fun create(x) = ref [x];

    fun push(x,s) = s := x::(!s);

    fun top(ref nil) = raise EmptyStack
    |   top(ref(x::xs)) = x;

    fun pop(ref nil) = raise EmptyStack
    |   pop(s) = s := tl(!s);
end;
```

a) Write a signature that will require the stack elements to be strings.

b) Rewrite the structure so that a stack is a list, not a reference to a list. Each of the functions `create`, `push`, and `pop` must return the new stack rather than modifying the stack as a side-effect.