CS109A Problem Set #2
Due 4:30 PM, 1/27/95

1) (15 points) What is wrong with the following inductive "proof"?

   $S(n)$: in a field with n horses, all have the same color

   Basis: $n = 1$. If there is one horse, then surely all horses have the same color.

   Induction: Suppose $S(n)$ is true for some $n \geq 1$ and consider a field with $n + 1$ horses. Remove one horse, say H_1, from the field. Now there are n horses, and by the inductive hypothesis, all have the same color. Next, bring back H_1 and remove another horse, say H_2. Again there are n horses, and all have the same color by the inductive hypothesis. Since H_1 is now in the field, he/she has the same color as the first n horses, and therefore all $n + 1$ horses have the same color.

2) (15 points) Write an ML function `cycle` that takes a list $[a_1, a_2, \ldots, a_n]$ and moves the first element to the rear; i.e., it produces $[a_2, a_3, \ldots, a_n, a_1]$. The empty list should yield the empty list as a result.

3) (10 points) Using `cycle` from Exercise 2, write a function `cycleMany` that takes a list L and a nonnegative integer i and cycles L i times.

4) (10 points) Give an example of a value of type

   ```
   ((int * int) * (string list) * real) * (real * string)
   ```

   (Exercise 4.5d, p. 33 of EMLP)

5) (15 points) Define recursively the set of even-parity strings, by induction on the length of the string. *Hint*: It helps to define two concepts simultaneously, both the even-parity strings and the odd-parity strings. (Exercise 2.6.7, p. 69, FCS)

6) (20 points) The *greatest common divisor* (GCD) of two integers i and j is the largest integer that divides both i and j evenly. For example, $gcd(24, 30) = 6$, and $gcd(24, 35) = 1$. Write a recursive ML function that takes two integers i and j, with $i > j$, and returns $gcd(i, j)$. *Hint*: You may use the following recursive definition of gcd. It assumes that $i > j$ (which you may **not** assume in your function).

   Basis: If j divides i evenly, then j is the GCD of i and j.

   Induction: If j does not divide i evenly, let k be the remainder when i is divided by j. Then $gcd(i, j)$ is the same as $gcd(j, k)$. (Based on Exercise 2.7.8, p. 74–75 of FCS)

7) (15 points) The function `sum`:

```
DefCell(int, CELL, LIST);

int sum(LIST L)
{
    if (L == NULL) return(0);
    else return(L->element + sum(L->next));
}
```

computes the sum of the elements on its given list (whose cells are of the usual type as defined by the macro `DefCell` of Section 1.6 and used in the merge-sort program of Section 2.8) by adding the first element to the sum of the remaining elements; the latter sum is computed by a recursive call on the remainder of the list. Prove that `sum` correctly computes the sum of the list elements. What statement $S(i)$ do you prove inductively? What is the basis value for i? (Exercise 2.9.2, p. 87 of FCS)