CS109A Mock Homework

This "homework" is not to be handed in. However, you are strongly encouraged to work these problems as if your life or grade (whichever you value more highly) depended on it. We shall distribute solutions, hopefully on 3/15; no later than 3/17.

1) (15 points) Let R be the relation defined by aRb, bRc, cRd, aRc, and bRd.

a) Draw the graph of R.
b) Is R a function?
c) Name two possible domains for R; name two possible ranges.
d) What is the smallest set S such that R is a relation on S (i.e., the domain and the range can both be S)? (Exercise 7.7.2, FCS)

2) (15 points) Let R be the relation on the set of integers $\{1, 2, \ldots, 10\}$ defined by aRb if a and b are distinct and have a common divisor other than 1. For example, $2R4$ and $6R9$, but not $2R3$.

a) Draw the graph for R.
b) Is R a function? Why or why not? (Exercise 7.7.4, FCS)

3) (15 points) Write an ML function `insert` that takes a pair of integers (x, y) and a relation R, and inserts (x, y) into the relation R. Assume that R is represented by a list of pairs; each pair consists of an integer a and a list of integers L such that aRb is true for exactly those b's on list L. Also assume that there are no duplicates on any list L, and that for each integer a in the current domain there is only one pair with a as the first component. Maintain these conditions after insertion; in particular, beware the situation where x does not appear as the first component of any pair.

4) (10 points) Give an example of a relation with declared domain $\{1, 2, 3\}$ that is reflexive and transitive but not symmetric. (Exercise 7.10.6(a), FCS)

5) (15 points) How many pairs are there in the relation $\subseteq_{\{1,2,3\}}$? In general, how many pairs are there in $\subseteq_U$, if U has n elements? *Hint*: Try to guess the function from a few cases like the two-element case (Fig. 7.27) where there are 9 pairs. Then prove your guess correct by induction. (Exercise 7.10.2, FCS)

6) (15 points) Using the datatype for general trees as in the 3/6 ML notes or Fig. 13.6 on p. 138, EMLP, write a function to list the labels of a tree in postorder. Your function must not use the concatenation operator, `@`. *Hint*: Use the "difference list" trick.

7) (15 points) Show that there is a 1–1 correspondence between the integers and the integral powers of 2, thus showing that the set of integral powers of 2 is countable. (Exercise 7.11.3(c), FCS)