CS109B Problem Set #1
Due 4:30 PM, Friday, 4/14/95

1) (15 points) Consider the graph of Fig. 9.5 (p. 458) to be an undirected graph, by replacing each arc $u \to v$ by an edge $\{u, v\}$.

   a) Find all the paths from a to d that do not repeat any node.

   b) How many simple cycles are there that include all six nodes? List these cycles.

   c) What are the neighbors of node a?

   (Exercise 9.2.2, FCS)

2) (20 points) In ML, we can represent a directed graph by a list of pairs. Each pair (v, L) is a node v and the adjacency list L for v. For simplicity, assume nodes are integers. Write a function that takes the representation of a directed graph and a pair of nodes (u, v), and tells whether there is an arc $u \to v$. *Note*: You may use "helper functions" if you like, but it isn't really necessary.

3) (15 points) Suppose that instead of merging lower trees into higher trees in the connected components algorithm of Section 9.4, we merge trees with fewer nodes into trees with a larger number of nodes. Is the running time of the connected-components algorithm still $O(m \log n)$? Why or why not? (Exercise 9.4.4, FCS)

4) (15 points) Use Kruskal's algorithm to find the MWST for each component of the graph with the following edges (Exercise 9.5.2, FCS):

CITY 1	CITY 2	DISTANCE
Marquette	Sault Ste. Marie	153
Saginaw	Flint	31
Grand Rapids	Lansing	60
Detroit	Lansing	78
Escanba	Sault Ste. Marie	175
Ann Arbor	Detroit	28
Ann Arbor	Battle Creek	89
Battle Creek	Kalamazoo	21
Menominee	Escanba	56
Kalamazoo	Grand Rapids	45
Escanba	Marquette	78
Battle Creek	Lansing	40
Flint	Detroit	58

5) (20 points) Prove by induction on k that a connected component with k nodes has at least $k - 1$ edges. (Exercise 9.4.2, FCS) *Note*: Careful! One may be tempted to start with a CC of k nodes and "push out" by adding a $k + 1$st node somewhere. That is not a valid form of inductive proof, because there is no reason to believe that you

thereby prove something about *every possible* CC of $k+1$ nodes (reread the box on p. 253, FCS). You need to start with a CC of $k+1$ nodes, remove a node somehow, claim that the result is a CC, and invoke the inductive hypothesis on the smaller CC.

6) (15 points) For the tree of Fig. 9.5 (p. 458, FCS), give two depth-first search trees starting at node a and one DFST starting at node d. For each tree, indicate the postorder numbers of the nodes. (Exercises 9.6.1 and 9.6.4, FCS)