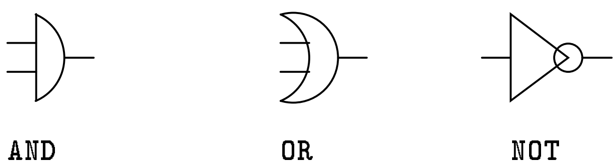
CS109B Notes for Lecture 5/24/95

Gates

No, not Bill — we mean a circuit element that implements a logical function.

- Gate inputs and outputs are usually voltages. We'll just use 0 and 1, assuming the gate can tell the difference.
- Here are symbols for 2-input `AND` and `OR` and 1-input `NOT`.
    - □ Inputs at left; output at right.
    - □ See Sect. 13.2 for other symbols.

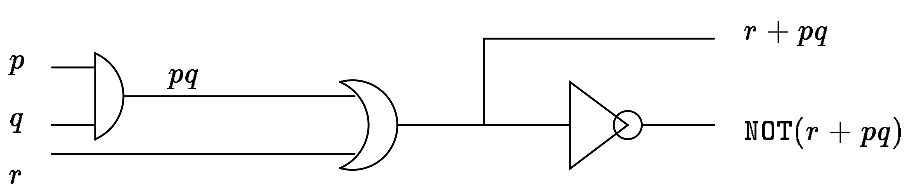


Circuits

Collection of gates wired together.

- A gate output can become an input to any number of gates.
- Some *circuit inputs* exist; these are not the output of any gate.
- One or more gate outputs are designated *circuit outputs*.

Example: Here is a circuit:



- Circuit inputs are p, q, and r.
- Circuit outputs at right.

From Circuits to Logical Expressions

If the circuit is *combinational* (= graph with gates as nodes, output→input connections as arcs has no cycles), then we can topologically sort the circuit.

- Then, we can visit the gates in this order and compute an expression for the gate output by applying the operator for that gate to the expressions at the gate inputs.

Example: For the previous circuit, the topological order is left-to-right.

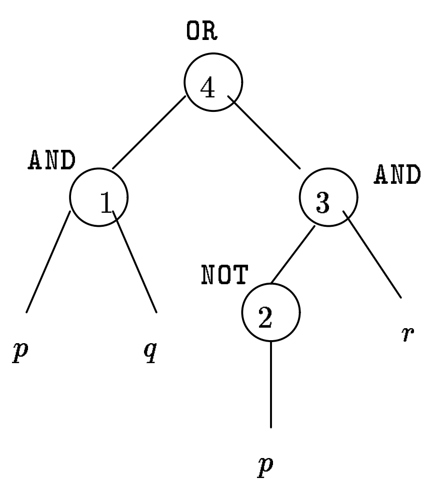
- The expressions associated with the gate outputs are indicated on the diagram.

From Logical Expressions to Circuits

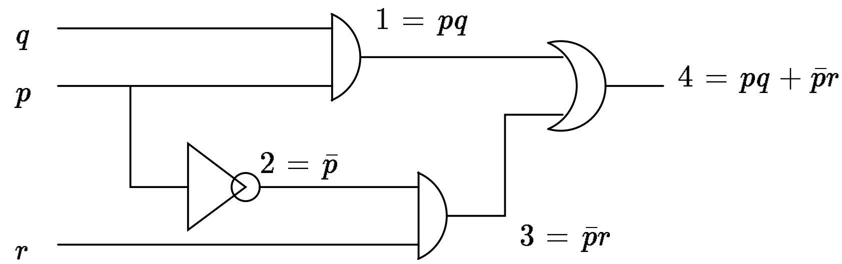
If we have gate types corresponding to all the operators of a logical expression, it is easy to build a combinational circuit for that expression.

- Create a gate for each subexpression.
- The type of a gate is the operand at the root of its subexpression.
- The inputs to a gate are the outputs of gates corresponding to the operands of the root.
  - ☐ If an operand is a variable, then that variable is a circuit input and also an input to the gate.

Example: Here is the expression tree for $pq + \bar{p}r$.



The following circuit has gates corresponding to the nodes of the tree.



Sequential Circuits

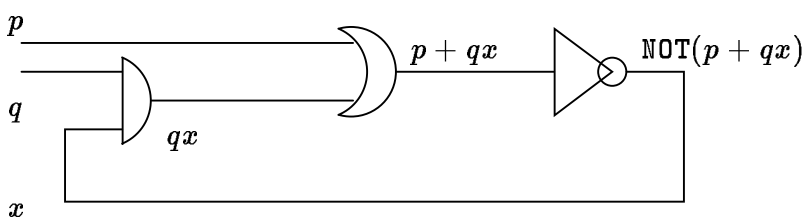
What if the circuit's graph has cycles (a *sequential* circuit)?

1. Introduce enough new variables so that every cycle contains at least one.

2. Express the new variables in terms of themselves and circuit inputs.

   - ☐ One of four things can happen to each variable: it remains 0, it remains 1, it oscillates between 0 and 1, or it becomes fixed at a value that depends on its initial value.

Example: Here is a sequential circuit:



Here is the truth table for the output $x = \texttt{NOT}(p + qx)$ in terms of the inputs p, q, and x.

p	q	x	$\texttt{NOT}(p + qx)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

- On inputs $p = q = 0$, whether x is 0 or 1 it becomes 1 as the gates operate.
- On inputs $p = 1$, $q = 0$ or 1, x becomes 0 regardless of its initial value.
- On inputs $p = 0$, $q = 1$ x oscillates. Its value at one instant is the opposite of its value at the previous "instant."

Class Problem

Analyze the behavior of the following sequential circuit.