Link-based clustering

- Given docs in hypertext, cluster into $k$ groups.
- Back to vector spaces!
- Set up as a vector space, with axes for terms as well as for in- and out-neighbors.

Example

```
1 2 3 4 ...
1 0 0 0 ...
```

Vector of terms in $d$

```
In-links  Out-links
1 3 4 5 ...
1 0 0 3 1 ...
```

In links Out links
Clustering

- Given vector space representation, run any of the clustering algorithms from lecture 8.
- Has been implemented on web search results.
- Other corpora: patents, citation structures.

Back up

- In clustering, we partition input docs into clusters.
- In *trawling*, we’ll enumerate subsets of the corpus that “look related”
  – will discard lots of docs
- Twist: will use purely link-based cues to decide whether docs are related

Trawling/enumerative clustering

- In hyperlinked corpora - here, the web
- Look for all occurrences of a linkage pattern
- Recall from hubs/authorities search algorithm:

Insights from hubs

- Link-based hypothesis:
- Dense bipartite subgraph \(\Rightarrow\) web community.
Communities from cores

- not easy, since web is huge
- what is a “dense subgraph”?
- define \((i,j)\)-core: complete bipartite subgraph with \(i\) nodes all of which point to each of \(j\) others

Fans                        Centers

(2,3) core

Random graphs inspiration

Every “large” enough “dense” bipartite graph “almost surely” has “non-trivial” core
e.g.:
- large = 3 by 10
- dense = 50% edges
- almost surely = 90% chance
- non-trivial = 3 by 3

Approach

- Find all \((i,j)\)-cores \((3 \leq i \leq 10, 3 \leq j \leq 20)\).
- Expand each core into its full community.

Finding cores

- “SQL” solution: find all triples of pages such that intersection of their outlinks is at least 3? Too expensive.
- Iterative pruning techniques actually work!
Initial data & preprocessing

- Crawl, then extract links
- Work with potential fans: nodes with $\geq j$ non-nepotistic links
- Eliminate mirrors
- Represent URLs by $2 \times 32 = 64$-bit hash
- Can sort URL’s by either source or destination using disk-run sorting

Popular page elimination

- Don’t want “popular” communities (Yahoo!, Excite, DejaNews, webrings, …)
- Popular community has popular page(s)
- Define popular page: indegree $\geq 50$

Main requirements

- Main memory conservation
- Few disk passes over data

Simple iterative pruning

- Discard all pages of in-degree $< i$ or out-degree $< j$.
- Repeat
- Reduces to a sequence of sorting operations on the edge list
Elimination/generation pruning

- pick a node $a$ of degree 3
- for each $a$ output neighbors $x$, $y$, $z$
- use an index on centers to output in-links of $x$, $y$, $z$
- intersect to decide if $a$ is a fan
- at each step, either eliminate a page ($a$) or generate a core

Results after pruning

- Elimination/generation pruning yields >100K non-overlapping cores for small $i,j$.
- 5M unpruned edges
  - small enough for post-processing by a priori
  - build $(i+1, j)$ cores from $(i, j)$ cores

Exercise

- Work through the details of maintaining the index on centers to speed up elimination-generation pruning.

Exercise

- Adapt the a priori algorithm to enumerating bipartite cores.
Results for cores

Sample cores

- hotels in Costa Rica
- clipart
- Turkish student associations
- oil spills off the coast of Japan
- Australian fire brigades
- aviation/aircraft vendors
- guitar manufacturers

From cores to communities

- Use hubs/authorities algorithm without text query - use fans/centers as samples
- Augment core with
  - all pages pointed to by any fan
    - all pages pointing into these
  - all pages pointing into any center
    - all pages pointed to by any of these

Using sample hubs/authorities
Recommendation Systems

Recommendation systems

Recommendation Systems

Recommendation docs to user based on user’s context (besides the docs’ content).

Other applications:
- Re-rank search results.
- Locate experts.
- Targeted ads.
Input
Past transactions from users:
- which docs viewed
- which products purchased
- pages bookmarked...
- explicit ratings (movies, books...)

Current context:
- browsing history
- search(es) issued

Explicit profile info:
- Role in an enterprise
- Demographic info
- Interest profiles

Example

<table>
<thead>
<tr>
<th>Users</th>
<th>Docs viewed</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>d1, d2, d3</td>
</tr>
<tr>
<td>U2</td>
<td>d1, d2</td>
</tr>
</tbody>
</table>

Recommend d3 to U2.

Expert finding

In an enterprise setting, recommend U1 to U2 as an expert.

Simple Algorithm

U viewed d1, d2, d5.

Look at who else viewed d1, d2 or d5.

Recommend to U the doc(s) most “popular” among these users.
More formally

\[ A = U \]

\[ A^T D A_{ij} = 1 \text{ if user } i \text{ viewed doc } j. \]

\[ AA^T : \text{Entries give } \# \text{ of docs viewed by pairs of users.} \]

Voting Algorithm

- Row \( i \) of \( AA^T \): Vector whose \( j^{th} \) entry is the \# of docs viewed by both \( i \) and \( j \).
- Call this row \( r_i \), e.g., \((0, 7, 1, 13, 0, 2, \ldots)\)
- Then \( r_i^T A \) is a vector whose \( k^{th} \) entry gives a vote count to doc \( k \)
  - emphasizes users who have high weights in \( r_i \).
- Output doc(s) with highest vote counts.

What’s on the diagonal of \( AA^T \)?

Voting Algorithm - implementation issues

- Wouldn’t implement using matrix operations
  - use weight-propagation on data structures.
- Need to log and maintain “user views doc” relationship.
  - typically, log into database
  - update vote-propagating structures periodically.
- For efficiency, discard all but the heaviest weights in each \( r_i \).

What good was the matrix formulation?

\[ AA^T \] entries give us a similarity measure between users.
\[ r_i \] has proximities from user \( i \) to the rest.
\[ r_i^T A \] gives proximities from user \( i \) to the docs.
Need a more general formulation

- If a user is close to two docs d1 and d2, are the docs d1 and d2 close to each other?
- How do we combine different sources of content and context?
  - terms in docs
  - links between docs
  - users’ access patterns
  - users’ info.

Vector spaces again

Turn every entity into a vector.

Axes are terms, docs, user info …

e.g.,
  - Some axes for terms
  - One axis for each doc.
  - Additional axes for user attributes like gender, enterprise role, etc.

Vector Space

Each doc represented by \( tf \times idf \) weights for terms, plus a 1 entry for its own axis, and 0’s elsewhere.

Users represented by 1’s for docs viewed, 0’s elsewhere. User posing a query: \( tf \times idf \) weights for terms.

Context with content

- Docs’ content captured in term axes.
- Other attributes (user behavior, current query etc.) captured in other axes.
- A probe consists of
  1 : a vector \( v \) (say, a user vector plus a query)
  2 : a type of vector to be retrieved (say, a doc)
- Result = vectors of chosen type closest to \( v \)
### Implementation details

- Don’t really want to maintain this gigantic (and sparse) vector space
- Dimension reduction
- Fast near neighbors (of vectors from a given type)
- Incremental versions needed

### Resources