Problem Set 5

Problem 1. Perform the steps of iterative pruning to find (2,3) cores on the following graph.

Answer. The iterations are as follows:

Problem 2. Adapt the a priori algorithm to enumerating bipartite cores.

Answer. After all pruning steps are done, we can apply a priori to enumerate the remaining bipartite cores. Note that every subset of an \((i, j)\) core is itself an \((i', j')\) core, with \(i' \leq i, j' \leq j\). For each \(j\), initialize \(S\) to \((1, j)\) cores (i.e., all fans with \(\text{outdegree} \geq j\) and their neighbors. Now to enumerate \((2, j)\) cores, note that only those fans identified in \(S\) need to be considered when checking pairs of fans that point to \(j\) nodes. Likewise, when looking for \((i, j)\) cores, we need only consider fan subsets \(f\) of size \(i\) where each subset of \(f\) of size \(i - 1\) was identified in the previous iteration and placed in \(S\).

Problem 3. Referring to the Voting Algorithm slide (lecture 11, slide 34), what does the matrix \(AA^TA\) represent?

Answer. If \(A\) has dimensions \(u \times d\), then \(AA^TA\) is also a \(u \times d\) matrix, whose \(i\)'th row is a vector whose \(k\)'th entry gives a vote count to doc \(k\) wrt user \(i\).