THE ERRATA OF COMPUTER PROGRAMMING

by

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STAN-CS-79-712
January 1979

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This report lists all corrections and changes of Volumes 1 and 3 of The Art of Computer Programming, as of January 5, 1979. This updates the previous list in report CS551, May 1976. The second edition of Volume 2 has been delayed two years due to the fact that it was completely revised and put into the TEX typesetting language; since publication of this new edition is not far off, no changes to Volume 2 are listed here.

The present report was prepared with a typesetting system that is now obsolete; please do not wince at the typography. All cahnges and corrections henceforth will be noted in TEX form on file ERRATA.TEX[ART,DEK] at SU-AI.

In spite of inflation, the rewards to error-detectors are still $2 for "new" mistakes in the second edition, $1 in the first edition.

Please do not endanger the author's morale by asking him about Volume 4. Thank you for your understanding.

1.0 throughout the book(s) *2/28/78* 2

when the text of these books is on a computer I will try to be consistent in hyphenating compound adjectives like doubly-linked lists and storage-allocation algorithms, etc. . . . but until then, such lapses are not to be considered errors

1.2 line 1 1 *5/27/78* 3

Leibnitz ⇝ Leibniz

1.18 line -7 *11/29/77* 4

the theorem ⇝ that the theorem

1.18 line 16 *11/29/77* 5

3,... ⇝ **3,....**

1.35 line 3, under the big pi *11/12/76* 6

***n*,** ⇝ *n*

The preparation of this report was supported in part by National Science Foundation grant MCS-77-23738, by Office of Naval Research contract N00014-76-C-0330, and by IBM Corporation. Reproduction in whole or in part is permitted for any purpose of the United States government.

1.41 **displayed formula in exercise 32** *2/28/78* *7*

n/ c ⇝ ***n/c***

1.44 **add a footnote (see p. v for style)** *4/19/77* *8*

line 3 after (1): book. ⇝ book.'
footnote for bottom of page: In fact, permutations are so important, Vaughan Pratt has suggested calling them "perms." As soon as Pratt's convention is established, textbooks of computer science will be somewhat shorter (and perhaps less expensive).

1.44 **lines -4, -5(twice), -7, -15, -16** *11/12/76* *9*

··· ⇝ . . .

1.45 **lines 3, 10, 11, 12, 21** *11/12/76* *10*

··· ⇝ . . .

1.50 **exercise 21 line 1** *7/31/76* *11*

Faa ⇝ Fai

1.51 **line 13** *2/28/78* *12*

manner ⇝ matter

1.52 **line 6 after Table 1** *8/25/76* *13*

Sxu-yuen ⇝ ***Sru-yiian***

1.56 **change in Eq. (17)** *11/12/76* *14*

-*r* ⇝ ***r*** **and** ***r*** ⇝ **-*r***

1.57 **Eq. (18)** *7/31/76 1 5*

$n \geq 0.$ ⇝ $n.$

1.57 **iine after (19)** *11/12/76 1 6*

$-r$ ⇝ r

1.66 **caption to Table 2, replace third line by;** *9/21/76 1 7*

see D. E. Barton, F. N. David, and M. Merrington, *Biometrika* 47 (1960), 439-445; 50 (1963), 169-176.

1.72 **line -4** *11/15/78 1 8*

$A_{n(k-1)}$ ⇝ $A_{n-1)(k-1)}$

1.79 **lines 8,9,10** *6/25/76 1 9*

Kepler, ... life. ⇝ Johann Kepler, 1611, who was **musing** about the **numbers** he **saw** around him [J. Kepler, *The Six-Cornered Snowflake* (Oxford: Clarendon Press, 1966), p. 21].

1.83 **line -7** *I J/29/77 20*

use same style script F in this line as in line -6 (six places)

1.90 **new generalized Eq. (29)** *8/25/76 2 1*

$(z/(e^z-1))^n = 1 - (1/(n-1))\left[{n \atop n-1}\right]z \bullet (1/(n-1)(n-2))\left[{n \atop n-2}\right]z^2 - \cdots = \Sigma_{k\geq 0}\ B_k^{(n)}z^k/k!.$ (29)

(convert this to usual format for displayed equations)

1.90 **update to previous correction number 25** *11/12/76 2 2*

to appear, ⇝ 75-77,

1.91 replace lines 1-3 by the following new copy *8/25/76* 23

The coefficients $B_k^{(n)}$ which appear in the last formula are called "generalircd Bernoulli numbers"; **Scction** 1.2.11.2 **examincs** them further in the important special **casc** $n = 1$. For **small** k, *we* **have** $B_k^{(n)}/k! = (-1)^k\left[{n \atop n-k}\right](n-k-1)!/(n-1)!$, **but when** $k \geq n$ this formula breaks down since it reduces to 0 times ∞. An analogous situation holds for the power series $(z/\ln(1+z))^n$, where the coefficient of e^k for $k < n$ is $\left\{{n \atop n-k}\right\}(n-k-1)!/(n-1)!$.

1.92 line -8 *7/31/76* 24

Faa $\rightsquigarrow$ Fai

1.98 caption, line 2 *7/31/76* 25

2.11 $\rightsquigarrow$ 2.10

1.103 line 3 *7/31/76* 26

Faa $\rightsquigarrow$ **Faà**

1.110 three lines after (12) *6/25/76* 27

$R_m \rightsquigarrow |R_m|$

1.111 line 8 *11/15/78* 28

mately 2 $\rightsquigarrow$ mately $(-1)^{1+k/2}2$

1.116 line -6 *11/29/77* 29

Analysis $\rightsquigarrow$ A crude analysis

1.116 line -6 and Eq. (22) *11/29/77* 30

$n^{n-1/2} \rightsquigarrow n^n$

1.117 line 5 *11/29/77* *31*

three $\rightsquigarrow$ two

1.118 exercise 5 *11/29/77* *32*

$n^{n-1/2} \rightsquigarrow n^{n}$

1.125 line 2 *1/16/77* *33*

is loaded. $\rightsquigarrow$ are loaded.

1.126 line 1 *1/16/77* *34*

The contents $\rightsquigarrow$ A portion of the contents

1.126 line 7 *1/16/77* *35*

is $\rightsquigarrow$ are

1.127 line -19 *11/15/78* *36*

Overflow may occur as in **ADD.** $\rightsquigarrow$ Same as **ADD** but with **-V** in place of V.

1.127 lines -18 through -13 *11/15/78* *37*

move this paragraph in **front** of the **SUB** definition on the preceding two **lines**

1.134 line -12 *4/19/77* *38*

MUL requires $\rightsquigarrow$ **MUL, NUM, CHAR** each require

1.137 box 05 *4/19/77* *39*

1 $\rightsquigarrow$ *10*

1.150 lines -10,-9,-8 *4/19/77 40*

CON ⇝ CON (4 times)

1.152 line 16 *11/29/77 41*

facilate ⇝ facilitate

1.156 stylistic corrections *6/14/77 42*

line 2: i.e. ⇝ e.g.
line 3: (*X* ⇝ (Here X
line 5: sun. ⇝ sun;
line *10:* (*E* ⇝ (This number *E*
line 22: the year ⇝ that the year

1.198 lines 19-21 *6/14/77 43*

An illustration...See also the book ⇝ See, for example, the book

1.224 line -11 *6/14/77 44*

F = 7 ⇝ F = 9

1.225 line -9 *6/25/76 45*

about 1946 ⇝ during 1946 and 1947

1.237 line -10 *12/19/76 46*

down an item ⇝ an item down
up the stack ⇝ the stack up

1.248 insert new paragraph after line 4 *4/19/77 47*

Further study of Algorithm C has been made by D. S. Wise and D. C. Watson, *BIT* **16** (1976), 442-450.

1.258 line 4 *9/21/76 4 8*

we ⇝ exercise **30** describes a somewhat more natural alternative, and we

1.270 new exercise *9/21/76 4 9*

30. [*17*] Suppose that queues are represented as in (12), but with an empty **queue** represented by F ← A and **R** ***undefined.*** What insertion and deletion **procedures** should replace (14) and (17)?

1.303 exercise 9 line 4 *3/ 2/77* 5 0

girl6 ⇝ women

1.325 line 8 *4/19/77 5 J*

otherwise. ⇝ otherwise, making the latter node the right son of NODE (Q).

1.332 new quote to insert *just* before Section 2.3.2 *1/16/77 5 2*

Binary or *dichotomous* *systems, although **regulated** by a **principle,** are among the most **artificial** arrangements that have ever been Invented.*

--WILLIAM SWAINSON, *A **Treatise** on the Geography and **Classification** of **Animals,** Sec. 250* (1835)

1.339 line 13 *6/25/76 5 3*

In all ⇝ Furthermore TYPE (W) is set appropriately, depending **on** x. In all

1.382 line 2 *12/19/76 5 4*

there is a man now living having ⇝ somebody now living has

1.398 line -1 *5/27/78 5 5*

with ⇝ than

1.406 line -2 *1/16/77* 56

as ⇝ informally as

1.406 line 11 *5/27/78* 57

-types ⇝ **-tuples**

1.406 line 18 *11/15/78* 58

Polya ⇝ **Pólya**

1.414 step A2 lines 2-4 *2/28/78* 59

unmarked, mark it, and if ⇝ unmarked: mark it and, if (twice)

1.420 lines 14-15 *9/21/76* 60

[See the . . . 372.) ⇝ An elaborate system which does this, and which also includes a mechanism for postponing operations on reference counts in order to achieve further efficiency, has been described by L. P. Deutsch and D. G. Bobrow in *CACM* **19** (1976), 522-526.

1.420 line 17 *11/29/77* 61

see ⇝ see N. E. Wiseman and J. O. Hiles, *Comp. J.* 10 (1968), 338-343,

1.437 line 18 *6/25/76* 62

For these reasons the ⇝ A contrary example appears in exercise 7; the point **is that** neither method clearly dominates the other, hence the simple

1.445 line 11 *1/16/77* 63

each with a random **lifetime,** ⇝ *each equally likely to be the next one deleted,*

1,446 new paragraph after line 6 *1/16/77 6 4*

Our assumption that each deletion applies to a random reserved block will be valid if the lifetime of a block is an exponentially-distributed random variable. On the other hand, if all blocks have roughly the same lifctimo, this assumption is **falsc;** John E. Shore has **pointcd** out that type A blocks tend to be "older" than type *C* blocks **whcn** allocations and deletions **tcnd** to have a somewhat first-in-first-out character, since a soqucnce of adjacent **rcserved** blocks tends to be in order from youngest to oldest and since the most recently allocated block is almost never type A. This tends to produce a smallar number of available blocks, giving even better performance than the fifty-percent rule would predict. [Cf. *CACM* 20 **(1977),** 812-820.)

1,448 line -9 *11/15/78 6 5*

areas $\rightsquigarrow$ areas of the same **size**

1,451 line 7 *1/16/77 66*

. $\rightsquigarrow$; John E. Shore, CACM **18(1975),433-440.**

1,451 yet anot her addition after line 7 *2/28/78 6 7*

. $\rightsquigarrow$; Norman **R.** Nielsen, CACM 20 **(1977),** 864-873.

1,454 exercise 28 *4/19/77 6 8*

line 2: 5; for $\rightsquigarrow$ 5. For
line 4: " $\rightsquigarrow$ The execution time is $2u$."

1,456 line 8 *6/25/76 6 9*

V-1.] $\rightsquigarrow$ V-l; and see especially also the work of Konrad Zuse, ***Berichte det Gesellschaft*** fiir ***Math. und Datenv.*** 63 (Bonn, **1972),** written in 1945, Zuso was the first to develop nontrivial algorithms that worked with **lists** of dynamically varying lengths.]

1,456 line -7 *12/19/76 7 0*

is divisible $\rightsquigarrow$ is not divisible

1,458 *6/25/76 7 1*

lines -15 thru -13: The A-l . . . code; $\leadsto$ The machine language for several early computers used a three-address code to represent the computation of arithmetic expressions;

lines -11 and -10: the A-l compiler language $\leadsto$ an extended three-address code

1,460 line 2 *3/ 2177 72*

The latter $\leadsto$ Weizenbaum's

1,463 several changes *12/19/76 7 3*

line 1: . $\leadsto$,

line 4: older $\leadsto$ other

new paragraph to be inserted after line 4:

A **related** model of computation was proposed by A. N. Kolmogorov as early as 1952. His machine essentially operates on graphs C, having a specially designated starting vertex v_0. The action at each step depends only on the **subgraph** C 'consisting of all vertices at distance $\leq n$ from v_0 in C, replacing G' in G by another graph $G'' = f(G')$, where C $''$ includes v_0 and the vertices v at distance exactly n from v_0, and possibly other vertices; the remainder of graph G is left unaltered, its components are attached to the vertices v at distance n as **before.** Here n is a **fixed** number specified in advance for any particular algorithm, but it can be arbitrarily large. A symbol from a finite alphabet is attached to each vertex, and restrictions are made so that no two vertices with the same symbol can be adjacent to a common vertex. ***(See*** A. ***N.*** Kolmogorov, ***Uspekhi Mat. Nauk*** **8,4 (1953), 175-**176; Kolmogorov and **Uspenskiĭ,** ***Uspekhi Mat. Nauk*** **13,4 (1958),** 3-28, ***Amer. Math. Soc. Translations,*** **series** 2, 29 **(1963),** 217-245.) Such graph machines can easily simulate the linking automata defined above, taking one graph step per linking step; conversely, linking automata can simulate graph machines, taking at most a bounded **number** of steps per graph step when n and the alphabet size are fixed. The linking model is, of course, quite close to the operations available to programmers on real machines, while the graph model is not.

1,473 exercise 44 line 2 *11/12/76 7 4*

$x_k+y_i \leadsto x_j+y_k$

1,478 line 8 *1/16/77 7 5*

(to appear) $\leadsto$ **13 (1975), 251-261.**

1.482 line 1 — *7/31/76* 7 6

Faa ⇝ **Fai**

1.487 new answer, continued — *4/19/77* 7 7

For example, Eq. (6) holds for all complex k and n, except in certain cases when n is a negative intager; Eqs. (7), (9), (20) are never false, although they may occasionally take indeterminate forms such as $0\cdot\infty$ or $\infty+\infty$. We can even extend the binomial theorem (13) and Vandermonde's convolution (21), obtaining $\sum_k \binom{r}{a+k} x^{a+k} = (1+x)^r$ and $\sum_k \binom{r}{a+k}\binom{s}{b-k} = \binom{r+s}{a+b}$, formulas which hold for all complex r, s, l, a, b whenever the series converge, provided that complex powers are properly defined. [See L. Ramshaw, *Inf. Proc. Letters* 6 (1977), 223-226.)

1.487 new answer — *11/12/76* 7 8

42, $1/(r+1)\mathrm{B}(k+1, r-k+1)$, if this is defined according to exercise 41(b). In general it appears best to define $\binom{r}{k} = 0$ when k is a negative integer, otherwise $\binom{r}{k} = \lim_{s\to r} \Gamma(s+1)/\Gamma(k+1)\Gamma(s-k+1)$, since this preserves most of the important identities.

1.494 line 9 — *11/15/78* 7 9

Polya ⇝ **Pólya**

1.499 exercise 7 — *11/15/78* 8 0

(It is "Glaisher's constant" 1.2824271...) To ⇝ To
This formula . . . $n=4$. ⇝ (The constant A is "Glaisher's constant" 1.2824271..., which R. W. Cosper has proved equal to $(2\pi e^{\gamma-\zeta'(2)/\zeta(2)})^{1/12}$.)

1.500 exercise 5 — *11/29/77* 8 1

line 1: $2n-1$ ⇝ $2n+1$
line 2: has . . . dx. ⇝ changes sign at $r = n - O(\sqrt{n})$, so $R = O(\int_0^n |f''(x)dx) = O(|f'(r)|) + O(|f'(n)|) = O(f(n)/\sqrt{n})$.

1.502 exercise 17(b) line 6 — *3/ 2/77* 8 2

J2NN ⇝ J2P

1.502 exercise 19 *4/19/77* 8 3

24 $\rightsquigarrow$ 42
1+1)*u* $\rightsquigarrow$ **10+10)*u***

1.504 exercise 25 *4/19/77* 8 4

lines 11-12: operations" $\rightsquigarrow$ operations," jump6 on register even or odd, and binary shift6
last line: M. $\rightsquigarrow$ M, and others could set **register←rA, register←rX.**

1.504 *6/14/77* 8 5

line 1: 6 $\rightsquigarrow$ 5 (also make this change in previous correction no. 111)
line 6: 3494 $\rightsquigarrow$ 3495 and *6* $\rightsquigarrow$ **5**
line 7: 3495 $\rightsquigarrow$ 3496 and **5** $\rightsquigarrow$ 4
line 9: 3506 $\rightsquigarrow$ 3505 and *6* $\rightsquigarrow$ **5**
line 10: 16 $\rightsquigarrow$ 14

1.511 changes to answer 14 *6/14/77* 8 6

line 1: uses as much $\rightsquigarrow$ due in part to J. Petolino uses a lot of
line 2: as possible, in $\rightsquigarrow$ in
line 9: I NCX 1 $\rightsquigarrow$
line 10: G $\rightsquigarrow$ GMINUS1
lines -17 to end of **page, replace** by:

```
              INCA 61
              STA  CPLUS60
              MUL  =3//4+1=
              STA  XPLUS57(1:2)
CPLUS60       ENTA *
              HUL  =8//25+1=           rA = i? + 24
GMINUS1       ENT2 *                   E5.
              ENT1 1,2                 rI1 = C
              INC2 1,1
              INC2 0,2
              INC2 0,1
              INC2 0,2
              INC2 773,1               r12 = 11C + 773
XPLUS57       INCA -*,2                rA = 1 1C+Z-X+20+24·30 (≥ 0)
```

1.512 more changes to answer 14 *6/14/77 8 7*

delete the bottom line and replace lines 1-31 by:

```
            SRAX 5
            DIV  =30=             rX = E
            DECX 24
            JXN  4F
            DECX 1
            JXP  2F
            JXN  3F
            DEC1 11
            J1NP 2F
3H          INCX 1
2H          DECX 23               E6.
4H          STX  20MINUSN(0:2)
            LDA  Y                EC.
            MUL  =1//4+1=
            ADD  Y
            SUB  XPLUS57(1:2)     rA . D-41
20MINUSN    ENN1 *
            INCA 67,1             E7.
            SRAX 5                rX = D + N
            DIV  =7=
            SLAX 5
            DECA -4.1             rA = 31 - N
            JAN  1F               E8.
            DECA 31
            CHAR
            LDA  MARCH
            JMP  2F
1H          CHAR
            LDA  APRIL
```

1.513 new answer *6/14/77 8 8*

15. The first such year is A.D. 10317, although the error *almost* leads to failure in *A.D.* 10108+19k for $0 \le k \le 10$.

1.513 still more changes to answer 14 *6/14/77 8 9*

replace **lines 1-6** by:

```
BEGIN    ENTX 1950          "driver"
         ENT6 1950-2000       routine,
         JMP  EASTER          uses the
         INC6 1               above
         ENTX 2000,6          subroutine.
         J6NP EASTER+1
```

1.514 line 18 *11/29/77 9 0*

time. ⇝ time. (It would be faster to calculate $r_n(1/m)$ directly when m is small, and then to apply the suggested procedure.)

1.515 bottom line *11/29/77 9 1*

Berk'ly ⇝ Berkeley

1.516 lines -4,-3 *4/19/77 9 2*

3)+7 ⇝ 7.5)+16

1.517 exercise 12 lines 7-10 *5/27/78 9 3*

delete "Thus, . ..(b)."

1.518 line 5 *5/27/78 94*

19-27. ⇝ 19-27; E. G. Cate and D. W. Twigg, *ACM Trans. Math. Software* 3 (1977), 104-l 10.

1.546 new answer *9/21/76 9 5*

30, To insert, set P ⇐ AVAIL, INFO (P) ← Y, LINK (P) ← A, if F = A then F ← P else LINK (R) ← P, and R ← P. To delete, do (9) with F replacing T.

1,550 exercise 18 — 3/ 2177 96

denotes, ... are included. ⇝ denotes **"exclusive** or." Other invertible operations, such a6 addition or subtraction moduio the pointer field size, could also be used. It is convenient to include

1,550 exercise 2 — 3/ 2177 97

line 2: next . . . **list** point ⇝ next, **so** the **links** in the list must point

line 3: So . . . the ⇝ Deletion at both ends therefore implies that the

line 4: ways. ⇝ ways, On the other hand, exercise 2.2.4-18 shows that two links can be represented in a single link field; in this way general deque operations are possible.

1,553 exercise 9 step G4 — 3/ 2/77 9 8

desired **girls,** ⇝ young **ladies** desired,

1,558 line -6 — 5/27/78 9 9

"pedigrees", ⇝ **"pedigrees,"**

1,575 exercise 12 line 5 — 9/21/76 1 0 0

∞. ⇝ **∞.** Hore $c(i,j)$ mean6 $c(j,i)$ if $j < i$.

1,583 answer 5 — 1/ 5/79 101

There is . . . **exist.** ⇝ When n>l, the number of series-parallel network6 with n edge6 is $2c_n$ [see P. A. **MacMahon,** *Proc. London Math. Soc.* 22 **(1891), 330-339].**

1,588 fourth line before exercise 33 — 5/27/78 1 0 2

minimal. ⇝ minimal. [This argument in the case of binary tree6 was apparently first discovered by C. S. **Peirce** in an unpublished manuscript; **see** his ***New Elements of Mathematics*** 4 (The Hague: Mouton, **1976), 303-304.]**

1,594 **updates to previous change number 150** *9/21/76* **103**

to appear, ⇝ 491-500,
(see also the important new contribution by H. G. Baker, Jr., *CACM* 21 (1978), 280-294, **for** which I will probably want to revise Section 2.3.5 entirely!)

1,594 **update to previous change number 151** *11/29/77* 104

Clark's list-copying algorithm appeared in ***CACM*** **21 (1978),** 351-357, and **Robson's** in CACM 20 (1977), 431-433

1,597 **last line of answer 6** *1/16/77* *105*

list. ⇝ list. For an alternative improvement to Algorithm A, **see** exercise 6.2.3-30.

1,597 **exercise 8** *6/25/76* **106**

line 1: also set R ⇝ also set $M \leftarrow \infty$, R
line 3: If R = A or tl ⇝ If M

1,601 **exercise 26 line 3** *2/28/78* **107**

two. ⇝ **two,** with blocks in decreasing **order of size.**
$P \geq M$ ⇝ $P \geq M - 2^k$.

1,601 **program line number 12** *4/19/77* **108**

j ⇝ *j*.

1,602 **new answer** *2/28/78* **109**

31. Seo David L. Russell, *SIAM J. Computing* 6 (1977), 607-621.

1,603 **addition to previous change 153** *4/19/77* *110*

.] ⇝ ; **Lars-Erik** Thoreiii, ***BIT*** 16 (1976), 426-441.

1.606 **exercise 41, numerator in value of a[5]** *6/14/77* *111*

19559 ↝ 18535

1.617L *6/25/76* *112*

delete A-l compiler, 458.

1.617L Aardenne-... *11/29/77* *113*

Taniana ↝ Tatyana

1.617R *12/19/76* *114*

AMM ↝ *AMM*

1.618L *5/27/78* *115*

Baker, Henry **Givens,** Jr., 594.

1.618R *4/19/77* *116*

add **p487** to entry for Binomial theorem, **generalizations of**

1.619L Bobrow entry *9/21/76* *117*

add **p420**

1.619R *5/27/78* *118*

Cate, Esko George, 518.

1.619R *11/29/77* *119*

Cheney, Christopher John, 420.

1.620R new definition entry — *12/19/76* *1 2 0*

Data organization: A way to represent information **in a data structure,** together with algorithms that access and/or modify this structure.

1.621L — *2/28/78* *1 2 1*

Derangements, **177.**

1.621L Deut sch entry — *9/21/76* *1 2 2*

add **p420**

1.622L End of file entry — *3/ 2/77* *1 2 3*

224 ↝ 223

1.623R Garwick entry — *11/15/78* *1 2 4*

244 ↝ 245

1.624L Hopper entry — *6/25/76* *1 2 5*

255,458. ↝ 225.

1.624L — *11/29/77* *1 2 6*

Hiles, John Owen, 420.

1.624R — *3/ 2/77* *1 2 7*

Invert a linked **list,** 266, 276.

1.624R INT entry — *6/14/77* *1 2 8*

225. ↝ 224-225.

1.625R *5/27/78* 129

Leibnitz (= Leibniz) $\leadsto$ **Leibniz (= Leibnitz)**

1.625R *12/19/76* 130

Kolmogorov, **Andreĭ** Nikolaevich, 463.

1.626R MacMahon entry *1/5/79* 131

add **p.** 583

1.627L *9/21/76* 132

Merrington, Maxine, 66.

1.628L *2/28/78* 133

Nielsen, Norman Russell, 451.

1.628R *5/27/78* 134

Peirce, Charles Santiago Sanders, 588.

1.629 *4/19/77* 135

add **p44** to Pratt entry

1.629L *6/14/77* 136

Petolino, Joseph Anthony, Jr., 511.

1.629R *5/27/78* 137

Prüfer, Heinz $\leadsto$ **Prüfer,** Ernst Paul Heinz

1.629R *6/25/76 138*

Prinz, Dietrich G.

1.630L *4/19/77 139*

Ramshaw, Lyle Harold, 487.

1.630R *3/ 2/77 140*

Reversing a list, 266, 276.

1.631L new entry *1/ 5/79 141*

Series-parallel networks, 583.

1.631L *1/16/77 142*

Shore, John E., 446, 451.

1.631L *2/28/78 143*

Russell, David **Lewis,** 602.

1.632L *1/16/77 144*

Swainson, William, 332.

1.632L Stirling numbers entry *8/25/76 145*

90, ↝ **90-91,**

1.632R *4/19/77 146*

add **p630** to **Thorelli** entry

1.633R *4/19/77 147*

Watson, Dan **Caldwell,** 248.

1.633R *4/19/77 148*

add **p487** to Vandermonde entry

1.633R *5/27/78 149*

Twigg, David William, 518.

1.633R van Aardenne-... *11/29/77 150*

Taniana ⇝ Tatyana

1.633R *12/19/76 151*

Uspenskiĭ, Vladimir Andreevich, 463.

1.634L *4/19/77 152*

add **p248** to Wise entry

1.634L *6/25/76 153*

Windley, Peter F.

1.634L Weizenbaum entry *9/21/76 154*

delete **p420**

1.634L *11/29/77 155*

Wiseman, Neil **Ernest,** 420.

1.634R *6/25/76 156*

Young Tanner, Rosalind Cecilia **Hildcgard, 75.**

1.636 (namely the endpapers of the book) *4/19/77 J57*

also make any changes specified for pages **136-137**

3.0X quotation for bottom of page *5/27/78 158*

Two hours' *daily exercise, . ,* ***will*** *be enough*
to keep a hack Fit For his work.
--M. H. MAHON, *The Handy Horse Book* **(Edinburgh, 1865)**

3.8L line 21 *3/ 2/77 159*

mädeln ⇝ Mädeln

3.8R line 26 *3/ 2177 160*

Weiner ⇝ Wiener

3.24 line 13 *2/28/78 16 J*

(1965 ⇝ **(1965)**

3.34 bottom line of determinant on line 12 *5/27/78 162*

a_{mn} ⇝ a_{mm}

3.40 Eq. (26) *2/28/78 163*

the j **in** e^j should be in smaller (superscript size) font

3.57 line 2 of step S3 *2/28/78 164*

right ⇝ right of

3.58 line 4 — 2128178 165

$a_1\ a_2,$ ⇝ $a_1, a_2,$

3.63 line -4 — 5/27/78 166

s's ⇝ X's
x's ⇝ s's

3.65 line -8 — 2/28/78 167

to better **understand** t_n ⇝ to understand t_n better

3.67 following (50) — 5/27/78 168

lines 2-4: we **find...Euler's** ⇝ Euler's
line 5: in this **case,** since ⇝ since
lines 7-8 (the two lines following (51)): n**; this...we** have proved that ⇝
n. The derivative $g^{(m)}(y)$ is a polynomial in y time6 e^{-2y^2}, hence R_m • $O(n^{(t+1-m)/4})$ $\int_{-\infty}^{+\infty}|g^{(m)}(y)|\,dy = O(n^{(t+1-m)/4})$. Furthermore if we replace α and β by $-\infty$ and $+\infty$ in the right-hand **side** of (SO), we make an error of at most $O(\exp(-2n^{\epsilon}))$ in each term. Thus

3.69 exercise 8 — 6/14/77 169

accent over o in **Erdös** should be " not ¨

3.72 new copy for exercise 28 — J1/15/78 170

28. [Mb33 Prove that the average length of the longest increasing subsequence of a random permutation on (1, 2, ... n} is asymptotically $2\sqrt{n}$. (This is the average length of row 1 in the correspondence of Theorem A.)

3.79 last line before exercises — 9/21/76 171

Feurzig ⇝ Feurzeig

3.83 lines 7 and 12 — 11/29/77 172

$\log_2$ ⇝ lg

3.98 line 4 J 1/29/77 173

$\log_2 \rightsquigarrow \lg$

3.104 line -2 6/14/77 174

inversions. $\rightsquigarrow$ inversions. Discuss corresponding improvements to Program S.

3.117 simplifications of step Q2 12/19/76 175

line 3: $K \leftarrow K_l, R \leftarrow R_l. \rightsquigarrow K \leftarrow K_l.$
line 4: K and $R \rightsquigarrow K$

3.118 comment to program line 05 12/19/76 176

$K \leftarrow K_l,\ R \leftarrow R_l. \rightsquigarrow K \leftarrow K_l.$

3.120 line -3 6/14/77 177

$\mathrm{s}_N \rightsquigarrow s_N$

3.122 line -6 12/19/76 178

instructions "$K \leftarrow K_l,\ R \leftarrow R_l$" $\rightsquigarrow$ instruction "$K \leftarrow K_l$"

3.128 line -3 4/19/77 179

$v.$ $\rightsquigarrow$ $v.$ Yihsiao Wang has suggested that the mean of three key values such as (28) be used as the threshold for partitioning; he has proved that the number of comparisons required to sort uniformly distributed random data will than be asymptotic to 1,082 $n \lg n$.

3.132 10 lines after (42) 5/27/78 180

$(N/x)^t \rightsquigarrow (N/xe)^t$

3.132 7 lines after (42) . 5/27/78 181

$O(N^{t-1/2}e^{-\pi N/2}) \rightsquigarrow O(|t+iN|^{t-1/2}e^{-t-\pi N/2})$

3.133 **in the discussion following (45)** *5/27/78* 182

line 3: $N^t \leadsto |M+iN|^t$
line 4: negligible. $\leadsto$ negligible, when N and N' are much larger than M.

3.134 **Eq. (46) and the line following** *2/28/78* 183

, $\leadsto$ $+\ O(n^{-M})$,
where $\leadsto$ for arbitrarily large M, where

3.134 **displayed formula on line 12** *2/28/78* 184

$f(n) \leadsto |f(n)|$
1725 $\leadsto$ 173

3.135 **exercise 16** *11/29/77* 185

HM46 $\leadsto$ *HM42*

3.138 **exercise 46 lower limit of integral** *6/14/77* 186

$a+i\infty \leadsto a-i\infty$

3.138 **exercise 52 binomial coefficient in the sum** *6/14/77* 187

remove spurious fraction line between 211 and $n+t$

3.144 **line 10** *2/28/78* 188

Language, $\leadsto$ *Language*

3.153 *11/12/76* 189

about here I will someday insert material about the new "binomial queue" algorithms to be discussed in papers by Vuillemin and Brown, since they appear to outperform leftist trees

3,158 **line -5** *5/27/78* 190

$a_i \rightsquigarrow a_1$

3,167 **line 21 of program** *5/27/78 191*

$L_q \rightsquigarrow L_p$

3,176 **line -12** *5/27/78 192*

$M{\cdot}b \rightsquigarrow M{\cdot}b^r$

3,177 **lines 25-27** *9/21/76 193*

that the multiplicity . . . Algorithm R, even $\rightsquigarrow$
that it ultimately spends too much time fussing with very small piles. Algorithm R is relatively efficient, even

3,192 **line -7** *5/27/78* 194

Well's $\rightsquigarrow$ Wells's

3,193 **line -15** *5/27/78 195*

less $\rightsquigarrow$ fewer

3,199 **Eq. (4)** *2/28/78 196*

$\lg \lceil \rightsquigarrow \lceil \lg$

3,208 **replacement for exercise 14** *11/29/77 197*

14. [*41*] (F. K. Hwang.) Let $h_{3k} = \lfloor (43/28)\cdot 2^k \rfloor - 1$, $h_{3k+1} = h_{3k} + 3\cdot 2^{k-3}$, $h_{3k+2} = \lfloor (17/7)\cdot 2^k - 6/7 \rfloor$ for $k \ge 3$, and let the initial values be defined so that $(h_0, h_1, h_2, \ldots) = (1, 1, 2, 2, 3, 4, 5, 7, 9, 11, 14, 18, 23, 29, 38, 48, 60, 76, 97, 121, 154, \ldots)$. Prove that $M(3,h_t) > t$ and $M(3,h_t - 1) \le t$ for all t, thereby establishing the exact values of $M(3,n)$ for all n.

3.215 bottom line of Table 1 *3/ 2/77 198*

1 7 $\rightsquigarrow$ **16**** (twice)
add footnote:
xx See K. Noshita, ***Trans. of the IECE of Japan,* E59,** 12 (Dec. **1976), 17-18.**

3.215 line 4 after second illustration *3/ 2/77 199*

the values listed in the table for $n \geq 8$ $\rightsquigarrow$ the values shown for $V_4(9)$, $V_5(10)$ **and their** duals $V_6(9), V_6(10)$

3.217 amendment to previous correction number 242 *12/19/76 200*

line 17: A. Schiinhage [to appear] $\rightsquigarrow$ A. Schiinhage, M. Paterson, and N. Pippenger [***J. Camp. Sys.*** Sci, 13 **(1976), 184-199]**

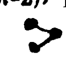
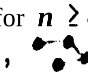
line 18: asymptotic $\rightsquigarrow$

lines 19-20: $3n$, and . . . $1.75n$. $\rightsquigarrow$ $3n + O(n \log n)^{3/4}$. On the other hand, Vaughan Pratt has obtained an asymptotic lower bound of $1.75n$ for this problem [cf. ***Proc.*** IEEE ***Conf. Switching and Automata Theory 14*** **(1973), 70-81];** a generalization of his result appears in exercise **25.**

3.219 exercise 14 *12/19/76 201*

Show that . . . comparisons. $\rightsquigarrow$ Let $U_t(n)$ be the minimum number of comparisons needed to find the t largest of n elements, without necessarily knowing their relative order. Show that $U_2(5) \leq 5$.

3.220 new exercise *12/19/76 202*

26. **[*M32*]** (A. **Schönhage,** 1974.) (a) In the notation of exercise 14, prove that $U_t(n) \geq \min(2+U_t(n-1), 2+U_{t-1}(n-1))$ for $n \geq 3$. ***Hint:*** Construct an adversary by reducing from n to n-l as soon as the current partial ordering is not composed of components • or •–•. **(b)** Similarly, prove that $V,(n) \geq \min(2+U_t(n-1), 3+U_{t-1}(n-1), 3+U_t(n-2))$ for $n \geq 5$, by constructing an adversary which deals with components •, •–•, , . **(c)** Therefore we have $U_t(n) \geq n+t+\min(\lfloor(n-t)/2\rfloor, t) - 3$ for $1 \leq t \leq n/2$. (d) The inequalities in (a) and (b) apply also when V or W replaces U, thereby establishing the optimality of several entries in Table 1.

5.225 line 1 *5/27/78* *203*

$\lfloor m/2 \rfloor$ ↝ $2\lfloor m/2 \rfloor$
$\lfloor n/2 \rfloor$ ↝ $2\lfloor n/2 \rfloor$

5.229 remarks about current best known sorting networks *1/16/77* *204*

line 19: D. Van Voorhis in 1974. ↝ R. L. Drysdale III in his undergraduate honors projeot at Knox College in 1973.
lines 20-21: a n lg $n + O(n)$ comparators, . ..3651. ↝
$(371/960)n$ lg n t O(n) comparators; in particular, his construction yields $\hat{S}(256) \le 3657$,
line 22: [To be published.] ↝ [*SIAM J. Computing* 4 (1975), 264-270.]

5.232 update to previous change number 250 *8/25/76* *205*

[*JACM*, to appear] ↝ [*JACM* 23 (1976), 566-571]

5.233 line 9 *5/27/78* *206*

)] ↝])

5.243 rating of exercise 48 *1/16/77* *207*

I I M49 ↝ *I I M46*

5.259 lines 4, 5, 6, 7 *9/21/76* *208*

has not yet . . . $m = 8$. This increase ↝
is difficult to analyze precisely, but T. 0. Espelid has shown how to extend the snowplow analogy to obtain an approximate formula for the behavior [*BIT* 16 (1976), 133-142]. According to his formula, which agrees well with empirical tests, the run length will be about $2P+b(m-1.5)(2P+b(m-2))/(2P+b(2m-3))$, when b is the block size and $m \ge 2$. Such an increase

5.260 insert new paragraph before Table 2 *2/28/78* *209*

The ideas of delayed run-reconstitution and natural selection can be combined, as discussed by T. C. Ting and Y. W. Wang in *Camp. J.* 20 (1977), 298-301.

3.262 line 7 *5/27/78* *210*

should be the square root of $(4e-10)P$

3.264 line -1 *5/27/78* *211*

beings $\rightsquigarrow$ begins

3.279 line 10 after Table 4 *6/14/77* *212*

JACM (to appear) $\rightsquigarrow$ *SIAM J. Computing* **6** (1977), 1-39

3.282 line before the big tableau *5/27/78* *213*

"*R*," $\rightsquigarrow$ "*R*",

3.284 line 22 *1/5/79* *214*

143 $\rightsquigarrow$ 145

3.284 lines 4, 13, 20 *1/5/79* *215*

25 $\rightsquigarrow$ 27

3.303 line -4 *8/25/76* *216*

always get $\rightsquigarrow$ always gets

3.326 line -7 *11/29/77* *217*

$L[p]$ $\rightsquigarrow$ $L[m]$

3.338 lines 1 and 7 *6/14/77* *218*

! $\rightsquigarrow$.

3.341 the foldout illustration *7/31/76 219*

in the bottom example (*10) look at line 4 of the six lines, where there is a longish black bar as the seventh activity (the sixth activity is a shorter black **bar)...and** lines **1,2,3,** and 5 have a blank bar just above and below this longish black bar; actually **lines 1,2,3,** and 5 should have parallel upward-slanting diagonal lines (the symbol for “reading in forward direction”) inside these blank bars

3.348 line 9 after the first illustration *5/27/78 220*

tape *C* ⇝ tape *A*
tape *D* ⇝ tape *B*

3.352 line -9 *6/14/77 221*

is ⇝ in

3.352 exercise 3 *11/29/77 222*

merge ⇝ radix sort

3.356 line -11 *5/27/78 223*

T3 ⇝ Track 3

3.358 line -20 *12/19/76 224*

artifically ⇝ tificially

3.370 Equation (8) *8/25/76 225*

B_2^2 ⇝ B_1^2

3.373 *6/25/76 226*

about **here** I should mention C. **McCulloch's** new approach to external disk sorting (embodied in the **KA** Sort on Honeywell 200)

3.374 st yiist ic improvements *1/16/77* 2 2 7

line 17: large, and . . . unthinkable! $\leadsto$ large; it is, however, so large that N seeks are unthinkable.
line 24: But $\leadsto$ On the other hand,
line 24: ! $\leadsto$.

3.381 table entries for Straight insertion *6/14/77* 2 2 8

Length: 12 $\leadsto$ 10
Space: N $\leadsto$ N + 1
Average: $2N^2+9N \leadsto 1.5N^2+9.5N$
Maximum: 4 $\leadsto$ 3
N-16: 494 $\leadsto$ 412
N=1000: 19855'74 $\leadsto$ 1491928

3.384 insert new paragraph before line -1 *6/25/76* 2 2 9

In Germany, *K.* Zuse independently constructed a program for straight insertion sorting in 1945, as one of the simplest examples of linear list operations in his "Plankalkül" language. (This pioneering work remained unpublished for nearly 30 years; see *Berichte dot Gesellschaft für Math. und Datenv.* *63* **(1972),** part 4, **84-85.)**

3.387 line 2 *8/25/76* 2 3 0

near-optional $\leadsto$ near-optimal

3.394 caption to Fig. 1 *3/ 2/77* 2 3 1

search. $\leadsto$ or "house-to-house" search.

3.394 Fig. 1 *4/19/77* 2 3 2

label the downward branch coming out of box S2 with an ■ sign

3.400 lines 12 and -5 *2/28/78* 2 3 3

running time $\leadsto$ average running time

3.412 **correction to previous change 263** *4/19/77* 2 3 4

delete this change, the book was right the first time

3.413 lines -4,-3 *4/19/77* *2 3 5*

and $N > 2^k$, we ⇝ we
$\lfloor \lg (N-2^k) \rfloor +1$ ⇝ $\lceil \lg(N+1-2^k) \rceil$

3.419 lines 13-14 *3/ 2/77* 2 3 6

H. Bottenbruch . . . He ⇝ D. H. Lehmer [*Proc. Symp. Appl. Math.* 10 (1960), 180-181] was apparently the first to publish a binary search algorithm which works for all N. The next step was taken by H. Bottenbruch [*JACM* 9 (1962), 214], who

3.419 line 30 *11/12/76* *2 3 7*

, but his flowchart and analysis were incorrect. ⇝ .

3.429 **line 7 (append to step D1)** *5/27/78* 2 3 8

(For example, if Q = RLINK (P) for some P, this means we would set RLINK (P) ← LLINK (T), etc.)

3.438 Fig. 16 *6/14/77* 2 3 9

insert "a)" and "b)" to the left of the roots of the trees, and change circles to squares in the right descendants of nodes AN and AS in the upper tree

3.439 **update to previous change 276** *11/15/78* 2 4 0

the Garsia-Wachs algorithm appeared in *SIAM J. Computing,* Dec. 1977, pp. 622ff; but now it seems an even better way has been found by Hu, Kleitman, and Tamaki (UCSD report 78-CS-016)

3.450 modifications to *exercise 33* *12/19/76* *2 4 1*

line 6: optimum. Cf. ⇝ optimum; cf.

line 7: .) ⇝ . On machines which cannot make three-way comparisons at once, a program for Algorithm T will have to make two comparisons in step **T2,** one for equality and one for less-than! B. Sheil and V. R. Pratt have observed that these comparisons need not involve the same key, and it may well be best to have a binary tree whose internal nodes specify an equality test or a less-than test but not always both. This situation would be interesting to explore as an alternative to the stated problem,)

3.451 line -3 *3/ 2/77* *2 4 2*

put a small inverted U over the ***ia*** in ***Akadamiia***

3.456 Fig. 22 *9/21/76* *2 4 3*

the arrows between boxes A2 and A3 should be reversed (downward arrow on left, upward arrow on right); also delete "P ← A" **below** boxes A3 and A4 and insert the words "Leaf found" between the two arrows leading to **A5**

3.457 line 15 *2/28/78* *2 4 4*

necessary. ⇝ necessary. Essentially the same method can be used if the tree is ***threaded*** (cf. exercise **6.2.2-2),** since the balancing act never needs to make difficult changes to thread links.

3.457 line after (4) *11/29/77* *2 4 5*

K ⇝ *K*

3.461 Table 1 *11/29/77* *2 4 6*

I will recompute this table, since **.144** should be ,143; also will modify the discussion on page 462 accordingly and will refer to **exercise** 11

3.461 line 2 after caption *11/29/77* *2 4 7*

change **+** and **-** to typewriter-style type (+ and -)

3.468 lines 6-9 *2/28/78* 248

I will rewrite this, as these trees have been studied almost too thoroughly by now

3.470 exercise 10 *11/29/77* 249

Does ... c? $\rightsquigarrow$ What is the asymptotic average number of comparisons **made** by Algorithm A when inserting the Nth item, assuming that items are inserted in random order?

3.470 exercise 16 *11/29/77* 250

the root node F were $\rightsquigarrow$ node E and the root node F were both

3.470 new exercise 11 *11/29/77* 251

[*M24*] (Mark **R.** Brown.) Prove that when $n \geq 6$ the average number of **external** nodes **of each** of the types +A, -A, **++B, +-B, -+B,** --B is exactly $(n+1)/14$, in a random balanced tree of n internal nodes constructed by Algorithm A.

3.472 near the bottom *11/15/78* 252

lines -7, -5, -4: log $\rightsquigarrow$ lg
line -3: 350 $\rightsquigarrow$ 307

3.479 update to previous change 293 *11/15/78* 253

, to appear $\rightsquigarrow$ 9 **(1978), 171-181**

3.479 new paragraph before the exercises *12/19/76* 254

It is possible for many indcpcndcnt users to bc accessing and updating different parts of a large *B*-**tree** file simultaneously without "deadlock," if the algorithms are implemented properly; see B. Samadi, ***Inf. Proc. Letters*** 6 **(1976), 107-112.**

3.483 line 25 *7/31/76* 255

55 $\rightsquigarrow$ **49**

3.486 lines 6 and -2 — *5/27/78* *256*

less ⇝ fewer

3.491 line -14 — *5/27/78* *257*

text, e.g. ⇝ text; e.g.,

3.505 line -14 — *5/27/78* *258*

to uniquely identify them ⇝ to identify them uniquely

3.507 line 13, add new sentence — *2/28/78* *259*

See R. Sprugnoli, *CACM* 20 (1977), 841-850, for a discussion of suitable techniques.

3.509 line 3 — *5/27/78* *260*

superimpose a / over the ■ sign

3.518 lines 5-7 — *4/19/77* *261*

using circular . . . complicated. ⇝ hashing FIRE and searching down its list, as suggested by D. E. Ferguson, since the lists are short.

3.526 new paragraph after line 19 — *11/29/77* *262*

E. G. Mallach *[Camp. J.* 20 (1977), 137-140] has experimented with **refinements** of Brent's variation, and further recent work on this topic has been done by G. Gonnet and I. Munro [*Proc.* ACM *Symp. Theory Comp.* 9 (1977), 113-121)

3.527 insertion of new material after line 20 *12/19/76* 263

Algorithm R may move some of the table entries, and this is undesirable if they are being pointed to from elsewhere. Another approach to deletions is possible by adapting some the ideas used in garbage collection (cf. Section 2.3.5): We might keep a "reference count" with each key telling how many other keys collide with it; then it is possible to convert unoccupied cells to empty status when their reference count is zero. Alternatively we might go through the entire table whenever too many deleted entries havc accumulated, changing all thc unoccupied positions to empty and then looking up all remaining keys, in order to sce which unoccupied positions really require 'deleted' status. This procedure, which avoids relocation and works with any hash technique, was originally suggested by T. Gunji and E. Goto [to appear].

3.528 update to previous change 307 11115178 264

[To **appear.]** ⇝ *J. Camp. Syst.* Sci. **16 (1978),** 226-214.

3.532 line after (48) *2/28/78* 265

likely we, ⇝ likely, we

3.534 line -5 *3/* 2177 266

buckote ⇝ pages or ***buckotr***

3.537 line -8 *4/19/77* 267

access ⇝ accesses

3.544 line 16 *6/14/77* 268

change one of ⇝ change

3.549 exercise 60 *1/5/79* 269

M48 ⇝ ***HM41***

3.549 another quote, put above the other *1/16/77* 270

She made a hash of the proper names, to be sure.
--GRANT ALLEN, *The Tents of Shem,* Ch. 26 (1889)

3.561 new paragraph to insert after line 18 *3/ 2/77* 271

If carefully selected nonrandom codes are used, it is possible to use superimposed coding without having any false drops, as shown by W. H. Kautz and R. C. Singleton, *IEEE Transactions* IT-10 (1964), 363-377; see exercise 16 for one of their constructions.

3.563 line 11 *5/27/78* 272

the N**D*E ⇝ the form N**D*E

3.563 line 9 *8/25/76* 273

his Ph. D. thesis (Stanford University, 1973).] ⇝
SIAM J. Computing 6 (1976), 19-50.]

3.566 Eq. (11) *3/ 2/77* 274

this is all wrong, it should be the 31 sextuples shown in the first printing of vol. 3 on page *565*

3.566 iine -7 *11/15/78* 275

Pfefferneuse ⇝ Pfefferneusse

3.570 line 6 *3/ 2/77* 276

systems or ⇝ systems on

3.570 new exercise *3/ 2/77* 277

16. [*25*] (W. H. Kautz and R. C. Singleton.) Show that a Steiner triple system of order v can be used to construct $v(v-1)/6$ codewords of u bits each such that no codeword is contained in the superposition of any two others.

3.576 new paragraph after answer 19 — *11/12/76* 2 7 8

A similar algorithm can be used to find $\max\{x_i+x_j \mid x_i+x_j \le c\}$; or to find, e.g., $\min\{x_i+y_j \mid x_i+y_j > t\}$ given t and two sorted files $x_1 \le \cdots \le x_m$, $y_1 \le \cdots \le y_n$.

3.576 line -6 — *12/19/76* 2 7 9

junctions; $\rightsquigarrow$ junctions; STELA, an alternative spelling of 'stele';

3.579 answer 7, line 3 — *5/27/78* 2 8 0

$>B_k$ and append (B_k+1) $\rightsquigarrow$ $\ge k-B_k$ and append $k-B_k$

3.585 new paragraph for answer 8 — *8/25/76* 28 1

A simple $O(n^2)$ algorithm to count the number of permutations of $\{1, \ldots, n\}$ having respective run lengths $l_1, \ldots, l_k$ has been given by N. G. de Bruijn, Nieuw *Archief voor Wiakunda (3)* 18 (1970), 61-65.

3.594 new answer — *11/15/78* 2 8 2

28. This result is due to A. M. Vershik and S. V. Kerov, *Dokl. Akad. Nauk SSSR* 233 (1977), 1024-1028. See also B. F. Logan and L. A. Shepp, Advancer in *Math.* 26 (1977), 206-222.

3.599 exercise 14 line 7 — *11/29/77* 2 8 3

13); $\rightsquigarrow$ 13), and still another by the identity in the answer to exercise 5.2.2-16 with $f(k) = k$;

3.603 exercise 33, comments to program — *7/31/76* 2 8 4

line 07: r12 $\rightsquigarrow$ r13
rI3 $\rightsquigarrow$ r 1 2
lines 09 and 15: To L4 $\rightsquigarrow$ To L4 with $q \leftrightarrow p$

3.604 replace lines 3 and 4 by the following new copy *6/14/77* 2 8 5

The ∞ trick also **speeds** up Program S; the following code **suggested** by J. H. Halperin use6 this idca and the MOVE instruction to reduce the running time to $(6B + 11N - 10)u$, assuming that location INPUT+N+1 already contain6 the **largest** possible one-word value:

```
01 START   ENT2 N-l        1
02 2H      LDA  INPUT.2    N-l
03         ENT1 INPUT, 2   N-l
04         JMP  3F         N-l
05 4H      MOVE 1,1(1)     B
06 3H      CMPA 1,1        B+N-1
07         JG   4B         B+N-1
08 5H      STA  0,1        N-l
09         DEC2 1          N-l
10         J2P  2B         N-l
```

Doubling up the inner loop would save an additional *B/2* or so unit6 of time.

3.605 exercise 4 *2/28/78* 2 8 6

lower **the** Σ sign and the relation below it

3.606 line 10 of the program *2/28/78* 2 8 7

r*A* ⇝ rA

3.606 answer 11 *11/29/77* 2 8 8

In general, . . . elements. ⇝ The situation becomes more complicated when N ≥ 64; R. Sedgewick has shown how to compute the worst-case permutations, and he has proved that the maximum number of exchanges is $1 - \lg\lg N / \lg N + O(1/\log N)$ times the number of comparisons *[SIAM J. Computing,* to appear].

5.607 new answer 16 *11/29/77* 289

16. Consider the $\binom{2n}{n}$ lattice paths from $(0,0)$ to (n,n) as in Figs. 11 and 18, and attach weights $f(i-j)$ if $i \geq j$, $f(j-i-1)+1$ if $i<j$, to the line from (i,j) to $(i+1,j)$; here f(k) is the number of bits $b_r \neq b_{r+1}$ in the binary expansion $k = (\ldots b_2 b_1 b_0)_2$. The total number of exchanges on the final merge when $N = 2n$ is
$\sum_{0 \leq j \leq i < n} (2f(j)+1) \binom{2i-j}{i-j} \binom{2n-2i+j-1}{n-i-1}$.
R. Sedgewick has simplified this sum to
$(1/2)n\binom{2n}{n} + 2\sum_{k \geq 1} \binom{2n}{n-k} \sum_{0 \leq j < k} f(j)$ and used the gamma function method to obtain the asymptotic formula $\binom{2n}{n}\left((1/4)n \lg n + (\lg(\Gamma(1/4)^2/2\pi)+1/4-(\gamma+2)/(4 \ln 2)+\delta(n))n + O(\sqrt{n} \log n)\right)$, where $\delta(n)$ is a periodic function of $\lg n$ with magnitude bounded by $.0005$; hence about $1/4$ of the comparisons lead to exchanges, on the average, as $n \to \infty$. *[SIAM J. Computing,* to appear.]

5.610 second line of answer 31 *11/29/77* 290

step ↝ **stop**

5.611 last line of answer 37 *2/28/78* 291

, ↝ .]

5.612 exercise 48 line 4 in limits to the integral *2/28/78* 292

$1/2$ ↝ $-1/2$ (twice)

5.616 line 26 of the program *2/28/78* 293

rA ↝ **rA**

5.618 answer 20 line 2 *5/27/78* 294

$0 \leq q < k$ ↝ $0 \leq q \leq k$

5.619 answer 27 line 1 *5/27/78* 295

$d \backslash n$ ↝ $d \backslash N$

5.627 line 16 — 1/ 5/79 296

See also ⇝ See also P. A. MacMahon, *Proc. London Math. Soc.* (1891), 341-344;

5.627 bottom of page, new paragraph for answer 6 — 8/25/76 297

M. Paterson observes that if the multiplicities of keys are $\{n_1, \ldots, n_m\}$, the number of comparisons can be reduced to $n \lg n - \Sigma n_i \lg n_i + O(n)$; see *SIAM J. Computing* **6** (1976), 2.

5.630 answer 20 — 5/27/78 298

line 5: $l-1$ ⇝ $l+1$
line 6: 2^{-l+1} ⇝ 2^{-l}
line 6: 2^{-l} ⇝ 2^{-l-1}
line 6: 2^{l} ⇝ 2^{l+1} (twice)
line 7: $\lfloor \lg N \rfloor + 1$ ⇝ $\lfloor \lg N \rfloor$

5.634 exercise 6 — 11/29/77 299

lg(...) ⇝ ⌈lg(...)⌉

5.635 answer 10 — 3/ 2/77 300

[*Inf. Proc. Letters* ⇝
]. ⇝ .

5.637 supplement to new answer 22 — 9/21/76 301

[See C. K. Yap, *CACM* **19** (1976), 501-508, for a further improvement,]

3,637 new answer 12/19/76 302

25. (a) Let the vertices of the two types of components be designated a; 6 < c. The adversary acts as follows on nonrcdundant comparisons: Case 1, $a{:}a'$, make an arbitrary decision. Case 2, $x{:}b$, say that $x >$ 6; all future comparisons $y{:}b$ with this particular 6 will result in y > 6, otherwise the comparisons are decided by an adversary for $U_t(n-1)$, yielding $\geq 2+U_t(n-1)$ comparisons in all. This reduction will be abbreviated "let 6 = min; $2+U_t(n-1)$." Case 3, $x{:}c$, let c = max; $2+U_{t-1}(n-1)$.

(b) Let the new types of vertices be designated $d_1, d_2 < e; f < g < h > i$. Case 1, $a{:}a'$ or $c{:}c'$, arbitrary decision. Case 2, $a{:}c$, say that a < c. Case 3, $x{:}b$, let b=min; $2+U_t(n-1)$. Case 4, $x{:}d$, let d = min; $2+U_t(n-1)$. Case 5, $x{:}e$, let e = max; $3+U_{t-1}(n-1)$. Case 6, $x{:}f$, let f = min; $2+U_t(n-1)$. Case 7, $x{:}g$, let f and g = min; $3+U_t(n-2)$. Case 8, $x{:}h$, let h = max; $3+U_{t-1}(n-1)$. Case 9, $x{:}i$, let i = min; $2+U_t(n-1)$.

(c) For $t=1$ we have U,(n) = n-l, so the inequality holds. For 1 < $t \leq$ n/2-1, use induction and (b). For $t = (n-1)/2$, use induction and (a). For t = n/2, $U_t(n-1) = U_{t-1}(n-1)$; use induction and (a). Exercise 14 now yields the following lower bound for the median:
$V_t(2t-1) \geq 3t+\lfloor t/2\rfloor-4$.

3,640 update to previous correction number 345 2/28/78 303

(To appear.) ⇝ *IEEE Trans.* **C-27** (1978), 84-87.

3,641 line -2 1/16/77 304

Pollard.] ⇝ Pollard.] All such identities can be obtained from a system of four axioms and a rule of inference for multivalucd logic due to Eukasicwicz; see Rose and Rosser, *Trans. Amer. Math. Soc.* **87** (1958), l-53.

3,641 exercise 43 3/ 2177 305

A. Waksman and M. Green have proved that ⇝ By slightly extending a construction due to L. J. Goldstein and S. W. Lcibhoiz, *IEEE Trans.* **EC-16** (1967), 637-641, one can show that $P(n) \leq P(\lfloor n/2\rfloor) + P(\lceil n/2\rceil) + n - 1$, hence
Eq. 5.3.1-3, cf. ... Green also has proved ⇝ Eq. 5.3.1-3; M. W. Green has proved (unpublished)

3,642 line 14 5/27/78 306

← ⇝ >

3.645 new paragraph after answer 10 *2/28/78 307*

One might complain that the algorithm compares KEY values that haven't been initialized. If such behavior is too shocking, it can be avoided by setting ail KEYs to 0, say, in step R1.

3.658 line 7 *5/27/78 308*

increase l by 1, set . . . . and return ⤳ set . . . . increase l by 1, and return

3.665 exercise 3 line 7 *11/12/76 309*

Trabb-Pardo ⤳ Trabb Pardo

3.671 exercise 2 *2/28/78 310*

line 1: RTAG ⤳ RTAG(Q)

line 2: RLI NK(P). ⤳ RLINK(P) and RTAG(P) ← +. In step T4, change the test RLINK(P) ≠ A to RTAG(P) ≠ +.

last line: .] ⤳ . Similar remarks apply with simultaneous left and right threading.)

3.673 tree illustration in answer 23 *11/15/78 311*

5 ⤳ 9

3.675 new answer 11 *11/29/77 312*

11. Clearly there are as many +A's as --B's and +-B's, when $n \geq 2$, and there is symmetry between + and −. If there are M nodes of types +A and -A, consideration of all possible cases when $n \geq 1$ shows that the next random insertion produces M-l such nodes with probability $3M/(n+1)$, otherwise it produces exactly $M+1$ such nodes. The result follows. [To be published.]

3.676 new answer to exercise 16 *11/29/77 313*

Delete E; Case 3 rebalancing at D. Delete G; replace F by G; Case 2 rebalancing at H; balance factor adjusted at K.

(a new illustration, in the same style as before, must be supplied now)

3.677 answer 20 — *8/25/76 314*

the line following the tree should **become** the following (instead of what was stated in the former correction number 355):
It is perhaps most difficult to insert a new node at the extreme left of a tree like this. An insertion algorithm taking at most $O(\log n)^2$ steps has been presented by D. S. Hirschberg, *CACM* 19 **(1976),** 4'71-473.

3.678 update to previous change 678 — *11/15/78 315*

, to appear $\rightsquigarrow$ 9 **(1978),** 171-181

3.679 changes to answer 5 — *6/14/77 316*

450. The worst . . . chars. $\rightsquigarrow$
Interpretation 1, trying to **maximize** the stated minimum: 450. (The worst . . . chars.)

Interpretation 2, trying to equalize the number of keys after splitting, in order to keep branching factors high: 155 (15 short keys followed by 16 long ones).

3.680 bottom, new paragraph for answer 4 — *7/31/76 317*

A more versatile way to economize on trie storage has been proposed by Kurt Maly, CACM 19 **(1976), 409-415.**

3.684 line -8 — *2/28/78 3 18*

$n \rightsquigarrow N$

3.687 exercise 1 — *2/28/78 3 19*

$-38 \rightsquigarrow -37$

3.687 answer 4 — *6/14/77 3 2 0*

change line 1 to: Consider cases with k pairs. The smallest n such that
in line 2 (the displayed formula), interchange m and n everwhere, then add ", for $m = 365$,"

3.687 update to previous change number 365 *6/14/77* *321*

Computing, to appear. $\leadsto$ *Computing 6* (1977), 201-234.

3.688 new answer *12/19/76* *322*

10. See F. **R. K.** Chung and R. **L.** Graham, ***Ars Combinatoria*** 1 **(1976), 57-76.**

3.689 exercise 14 *6/14/77* 3 2 3

line 2: keys $\leadsto$ all keys
line 12: until $\leadsto$ until TAG (P) = 1 and
line 12: points $\leadsto$ points (perhaps indirectly through words with TAG = 2)

3.693 replace all but first line of answer 37 by: *12/19/76* 3 2 4

$$M^N N S_N = \tfrac{1}{3}\sum \binom{N}{k_1,\ldots,k_M}\bigl(k_1(k_1-\tfrac{1}{2})(k_1-1)+\cdots+k_M(k_M-\tfrac{1}{2})(k_M-1)\bigr)$$
$$= \tfrac{1}{3}M\sum\binom{N}{k}(M-1)^{N-k}k(k-\tfrac{1}{2})(k-1)$$
$$= \tfrac{1}{3}MN(N-1)(N-2)\sum\binom{N-3}{k-3}(M-1)^{N-k}+\tfrac{1}{2}MN(N-1)\sum\binom{N-2}{k-2}(M-1)^{N-k}$$
$$= \tfrac{1}{3}MN(N-1)(N-2)M^{N-3}+\tfrac{1}{2}MN(N-1)M^{N-2}.$$

The variance is $S_N - ((N-1)/2M)^2 = (N-1)(N+6M-5)/12M^2 \approx \frac{1}{2}\alpha+\frac{1}{12}\alpha^2$.

3.698 new answer *1/ 5/79* 3 2 5

60. No; see M. Ajtai, J. **Komlós,** and E. **Szemerédi, *Inf. Proc. Letters* 7 (1978), 270-273.**

3.700 new answer *3/ 2/77* 3 2 6

16, Let each **triple** correspond to a codeword, where each codeword has exactly three 1 bits, **identifying** the elements of the **corresonding** triple. If u, v, w are distinct codewords, u has at most two 1 bits in common with the superposition of v and w, since it had **at** most one in common with v or w alone. [Similarly, from quadruple systems of order v we **can** construct $v(v-1)/12$ codewords, none of which is contained in the superposition **of** any **three** others, etc.)

3.703 update to previous correction number 373 *11/12/76* *3 2 7*

appear in the $\leadsto$ appear in **Eq.** 5.2.3-19 and in the

3.710L 1/5/79 3 2 8

Ajtai, Miklos, 698.

3.710L 1/16/77 3 2 9

Allen, Charles Grant Blairfindie, 549.

3.710L 4/19/77 3 3 0

add p576 to *AND* entry

3.711 11/15/78 331

delete index entries for R. M. Baer and P. Brock

3.711R 11/29/77 3 3 2

Brown, Mark Robbin, 470.

3.712L 4/19/77 3 3 3

delete Circular lists entry

3.712L 12/19/76 3 3 4

Chung, Fan Rang King, 688.

3.712R de Bruijn entry 8/25/76 3 3 5

add p. 585

3.712R 12/19/76 3 3 6

Deadlock, 479.

3.71 3 *6/14/77 3 3 7*

accent over o in Erdös should be " not . '

3.713L *J/16/77 338*

Drysdale, Robert Lewis (Scot), III, 229.

3.713R *4/19/77 3 3 9*

add p576 to Exclusive or entry

3.713R *J1/12/76 340*

Espelid, Terje Oskar, 259.

3.714L *4/19/77 3 4 1*

add p518 to Ferguson entry

3.714L line 5 *9/21/76 3 4 2*

Feurzig ⤳ Feurreig

3.714R *2/28/78 3 4 3*

Gonnet Haas, Gaston Henry, 526.

3.714R *3/ 2/77 3 4 4*

Goldstein, Larry Joel, 641.

3.714R *6/14/77 3 4 5*

Halperin, John Harris, 604.

3.714R *6/14/77 3 4 6*

h-ordered, 86-92, **103-104, *see*** %-ordered.
h-sorting, 86-92.

3.714R *11/29/77 3 4 7*

add **p607** to Gamma function entry

3.714R *12/19/76 3 4 8*

Goto, Eiichi, 527.

3.714R *12/19/76 3 4 9*

Cunji, Takao, 527.

3.715L *4/19/77 3 5 0*

Index mod ***p*, *9*.**

3.715L *9/2 1/76 35 J*

Hirschberg, Daniel Syna Moses, 677.

3.715R new entry *5/27/78 3 5 2*

Interchanging blocks of data, ***598*** (exercise **6),** 664 (exercise **3).**

3.716L *1/ 5/79 3 5 3*

Koml ós, J **ános,** 698.

3.716L Klei t man entry *2/28/78 3 5 4*

640 ⇝ ***639***

3.716L *3/ 2/77* 3 5 5

Lehmer, Derrick Henry, 419.

3.716L *3/ 2/77* 3 5 6

add pp. 561, **570** to Kau **tz** entry

3.716L *11/15/78* 3 5 7

Kerov, S. V., 594.

3.716R *1/16/77* 3 5 8

add **p641** to Eukasiewicz entry

3.716R *3/ 2/77* 3 5 9

Leibholz, Stephen W., 641.

3.716R *6/25/76* 3 6 0

Lozinski **ĭ,** Eliezer **Leonid** Solomonovich, 621.

3.717L MacMahon entry *1/ 5/79* 3 6 1

add **p.** 627

3.717L *7/31/76* 3 6 2

Maly, Kurt, 680,

3.717L *11/29/77* 3 6 3

Mallach, Efrem Cershon, 526.

3.717L *12/19/76* 3 6 4

add p. 637 to the entry for Median

3.717R *2/28/78* 3 6 5

Munro, James Ian, 526.

3.717R *5/27/78* 3 6 6

Mahon, Maurice **Hartland (=** Magenta), ix.

3.717R *6/14/77* 3 6 7

ROVE, 604.

3.718L *3/ 2/77* 3 6 8

add p.215 to Noshita entry

3.718L *4/19/77* 3 6 9

delete Newell entry

3.718L *12/19/76* 3 7 0

Nitty gritty ↝ Nitty-gritty

3.718R *4/19/77* 371

Packed data, 401.

3.718R new entry *5/27/78* 3 7 2

Pardo, ***see*** Trabb **Pardo.**

3.718R Paterson entry *8/25/76 373*

add p. 627

3.719L *11/15/78 374*

add **p.** 576 to Pollard entry

3.719R *1/16/77 375*

Rose, Alan, 641.
Rosser, John Barkley, 641.

3.719R *3/ 2/77 376*

Rearrangeable network, ***see*** Permutation network.

3.719R new entry *5/27/78 377*

Rotation of data, 598.

3.720L *11/29/77 378*

add pp. 606, 607 to Sedgewick entry

3.720L *12/19/76 379*

Samadi, Behrok h, 479.

3.720L *12/19/76 380*

add p. 220 to **Schönhage** entry

3.720R *3/ 2/77 381*

add pp. 561, 570 to Singleton entry

3.720R entry for SLB *8/25/76 3 8 2*

add p. 509

3.720R *12/19/76 3 8 3*

Sheil, Beaumont Alfred, 450.

3.721L *2/28/78 3 8 4*

Sprugnoli, R , **507.**

3.721R replacement for previous change 416 *1/ 5/79 3 8 5*

Szemer **édi,** Endre, 528,698.

3.721R *1/16/77 3 8 6*

Shanks, Daniel Charles, 575.

3.722L *2/28/78 3 8 7*

Ting, T. C., 260.

3.722L Threaded tree entry *2/28/78 3 8 8*

add **p457**

3.722L *11/12/76 3 8 9*

Trabb-Pardo ⇝ Trabb **Pardo**

3.722R *1/ 16/77 3 9 0*

delete **p229** from Van Voorhis entry

3.722R *2/28/78 3 9 1*

Wang, Y. W., 260.

3.722R *3/ 2/77 3 9 2*

Wiener, **Norbert,** 8.

3.722R *3/ 2/77 3 9 3*

delete **p641** from **Waksman** entry

3.722R *4/19/77 3 9 4*

Wang, Yihsiao, 128.

3.722R new name6 *6/25/76 3 9 5*

Venn, John **L.**
Windley, Peter F.

3.722R *11/12/76 3 9 6*

Yap, Chee-Keng, 637.

3.722R *11/15/78 3 9 7*

Vershik, Anatolii Moiseevich, 594,

3.723R *6/14/77 3 9 8*

2-ordered, 87, 103, 112, 135.

3.726 **(namely the endpapers of the book)** *4/ 19/77 3 9 9*

also make any changes specified for pages **136-137** of volume **1**

3.749L — 12119176 400

add **p.** 450 to Vaughan Pratt entry

3.765 addendum to previous change 324 — 11/15/78 401

John M. Pollard has discovered an elegant method for index computation in about $O(\sqrt{p})$ operations mod p, requiring very little memory, based on the theory of random mappings. **See Math.** *Comp. 32* **(1978),** 918-924, where he also suggests another method based on numbers $n_j = r^j$ mod p that have only small prime factors.

9.1 changes for the book Mariages Stables — 1/1/77 402

p12 line 18: Ac ⇝ *Aa*
p14 line 4: Ab ⇝ Bb
p18 line **-5:** B_i ⇝ B_j and A_i ⇝ A_j (four changes)
p18 line -4: b_i ⇝ b_j and a_i ⇝ a_j (four changes)
p18 line -3: a_n ⇝ a_k
p22 line -5, -4, -3: *d:* ⇝ *b*: *b*: ⇝ *c: c:* ⇝ *d:*
p32 line 6: exercises ⇝ **exercices**
p32 line -5 exercise ⇝ **exercice**
p35 illustration: delete arc from 4 of clubs to 8 of hearts
p38 line -11: C ⇝ *B*
p47 line 2: *Chsbyshav* ⇝ *Tchébichev*
p50 lines -12, -10, -3 and **p51** line 5: **Chebyshev** ⇝ **Tchébichev**
p52 line -6: c ⇝ C
p65 line -4: m ⇝ m
p66 line -10, denominator **of third term in sum:** n+l ⇝ *n-l*
p71 line 8: que R_A . ⇝ **que**
p74 line -1: X ⇝ x
p78 line-7: X ⇝ x
p78 line -4: $Q[A]$ ⇝ $Q[t]$
p86 line 10: fcmmes. ⇝ **femmes?**
p87 line -10: ZZ' ⇝ Zz'
p92 line -8: exercise ⇝ **exercice**
p93 **line 4: et** (*Aa, Bb, Cc* ⇝ **et** (*Aa, Bc, Cb*
p93 lines -6,-3,-2: crossed-out *e* should be crossed-out c
p95 line 3: $n!P_n$ ⇝ $n!p_n$
p95 line 9: Σ ⇝ Σ_i
p95 line -2: formula should **be preceded by (3)**
p95 line *-2:* $dx_2,\ldots,dx_n dy_1 dy_2,\ldots,dy_n$ ⇝ $dx_2\ldots dx_n dy_1 dy_2\ldots dy_n$

9.1 Changes for Surreal Numbers *1/16/77 403*

p86 lines 13-14 should say: $II(y,X_L,z), II(Y_R,x,z)$.

p86 line -2, change final comma to a period
p86 line -1, **delete** this line

p112 line -5: ***p.*** The $\rightsquigarrow$ ***p.*** *[See* his incredible book ***On Numbers*** and ***Games,*** **published** by Academic Press in **1976.]** The

p113 *Mathematik* $\rightsquigarrow$ ***Analysis***

THE TEX/METAFONT PROJECT.

WHAT HAS BEEN DONE:

Don Knuth has finished (and frozen) the implementation of TEX (the typesetting system) and is currently involved in the implementation of METAFONT (the font generator).

WHAT WE WANT TO DO:

We want to complement TEX / METAFONT with a suitable hardware environment, namely:

* An XGP type device that will provide hardcopy capabilities both for proofreading and for (medium quality) originals.

* A high resolution typesetting device for high quality originals.

* A high resolution CRT terminal, capable of displaying TEX output.

We also want to make the system widely available, thus it is needed to implement it in a more widespread language (PASCAL).

And finally we would like to try our hand in making TEX more interactive than what it is now. (This one is a tougher cookie.)

IF YOU ARE INTERESTED:

There are many things to be done. There are learning oportunfties. There are academic goodies (units, CS293 projects, etc). And there is also monies.

FOR MORE INFO:

Send a message to LTP, or call 74425 or 74377.