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A Rollback Interval for Networks with an Imperfect Self-Checking Property

by

John J. Shedletsky

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DIGITAL SYSTEMS LABORATORY

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ABSTRACT

Dynamic self-checking is a technique used in computers to detect a fault quickly before extensive data contamination caused by the fault can occur. When the self-checking properties of the computer circuits are not perfect, as in the case with self-testing only and partially self-checking circuits, the recovery procedure may be required to roll back program execution to a point prior to the first undetected data error caused by the detected fault.

This paper presents a method by which the rollback distance required to achieve a given probability of successful data restoration may be calculated. To facilitate this method, operational interpretations are given to familiar network properties such as the self-testing, secureness, and self-checking properties.

An arithmetic and logic unit with imperfect self-checking capability is analyzed to determine the minimum required rollback distance for the recovery procedure.

INDEX TERMS: checkpoint, rollback distance, rollback interval, self-checking, self-testing.

INTRODUCTION

Computer systems can be fault tolerant only if recovery from a fault detection can be successfully accomplished [l]. Recovery is defined [2] as the continuation of system operation with data integrity after a fault is detected. Data integrity must be restored by overcoming the data contamination that may have occurred prior to fault detection. The error-control provided by the self-testing and self-checking properties of logic networks can be an important asset to the recovery procedure [3,4].

A network is self-testing [3,5] for a fault if at least one normally applied input vector results in a detection of the fault. Coded outputs and code checkers are the usual means of detection. A network is secure for a fault if every incorrect output caused by the fault is detected. A network that is both self-testing and secure is self-checking.

A fault in a self-testing network is eventually detected, but undetected data errors due to the fault may be propagated prior to fault detection. The additional property of secureness in a self-checking network guarantees that no such propagation occurs, thereby simplifying the recovery process. The recovery algorithm in a self-checking standby computer system for example, need not validate the integrity of previously generated data. Program execution may continue immediately after initialization of the spare.

In some cases, a self-checking capability may be infeasible, or cost prohibitive in terms of hardware, software, or time redundancy.

The alternative is an imperfect-self-checking capability provided by networks that are self-testing for a fault, but not completely secure. Self-testing only and partially self-checking [6] networks provide this capability,

Program rollback may be necessary to restore data integrity when the self-checking capability of the system is imperfect. A successful data restoration can only occur when the program rollback is to a point prior to the first undetected data error.

The required rollback dietance for successful data restoration is critical in real-time control applications where commands, once issued, cannot be rescinded. In this case, the required rollback distance translates to a required output buffer delay. Application dependent requirements may place a bound on this delay, thereby constraining the attainable reliability,

This paper presents a model to quantitatively describe the rollback distance required for successful data restoration in imperfect-self-checking networks. The model is probabilistic so the rollback distance is a function of the desired probability of successful data restoration.

In an operational sense, the required rollback distance is the only property of interest for an imperfect-self-checking network. Classifying a property of a network with respect to a certain fault set reveals little about the operational behavior of the network if the fault set does not include every fault that is reasonably likely to occur. The familiar dynamic checking properties are re-defined to facilitate an operational approach to the analysis. The error latency model

[7,8] is basic to the analysis.

An example arithmetic and logic unit (ALU) with imperfect-self-checking capability [9] is analyzed in section 4. This ALU exemplifies the design techniques used in the Stanford SDC-16 computer. The required rollback distance is plotted as a function of instruction mix.

In the following a fault is a physical defect that changes the function realized by a network. An error occurs when data assumes an incorrect value due to a fault.

THE ROLLBACK INTERVAL OF A FAULT IN AN IMPERFECT-SELF-CHECKING NETWORK

In a system environment, an imperfect-self-checking network with a fault can respond to the application of an input vector in three mutually exclusive ways, denoted by C, D, and E:

C (correct response) - Correct data is generated and propagated by the network.

D (detection) - A fault is detected, initiating system recovery. No data is propagated.

E (error) - Incorrect data is generated or propagated undetected by the network.

A fault is detected by a checker that signals the appearance of a noncode output vector [10]. The detected fault may be internal to the network or it may be a fault in a predecessor network that causes the application of an improper input vector. In the latter case, the detected fault is viewed as a multiple fault sticking each input line of the network to the corresponding bit values of the improper vector.

The input vectors applied to a network in a computer are generated by a process too complex to describe in the general case. A reasonable approach is a probabilistic description. We assume "'successive input vectors are applied to a network randomly and independently. The probability of applying a given input vector is specified by an input-vector probability distribution, which is assumed known.

Definition. The conditional response probabilities c_i, d_i, and e_i of a fault Fi in a network are the conditional probabilities of the responses C,D, and E repectively, given that the fault Fi is active in the network:

$$c_i \equiv Pr[C|F_i] \qquad di \equiv Pr[D|F_i] \qquad e_i \equiv Pr[E|F_i] .$$

Every input vector with a nonzero probability of application belongs to the set of normal input vectors $\mathfrak{N}$. A fault Fi partitions the set $\mathfrak{N}$ into three subsets $\mathfrak{NC}_i$, $\mathfrak{ND}_i$, $\mathfrak{NE}_i$. An input vector X is a member of the set $\mathfrak{NC}_i$, $\mathfrak{ND}_i$, or $\mathfrak{NE}_i$ if the application of X to the faulty network elicits the responses C, D, or E respectively. The conditional response probability ci, d_i, or ei is equal to the sum of the probabilities of those vectors belonging to $\mathfrak{NC}_i$, $\mathfrak{ND}_i$, or $\mathfrak{NE}_i$.

$$c_i = \sum_{j:X_j \in \mathfrak{NC}_i} Pr[X_j] \qquad d_i = \sum_{j:X_j \in \mathfrak{ND}_i} Pr[X_j] \qquad e_i = \sum_{j:X_j \in \mathfrak{NE}_i} Pr[X_j] \qquad (2.1)$$

The conditional response probabilities depend on the input vector probabilities. The conditional response probabilities sum to 1.

Example 1: In the one-out-of-eight decoder [11] in Fig. 1, a fault detection occurs when $z_1 z_2$ = <0 0> or <1 1>. Assuming equally-likely input vectors, Table 1 lists the single stuck-at faults and their respective conditional response probabilities.

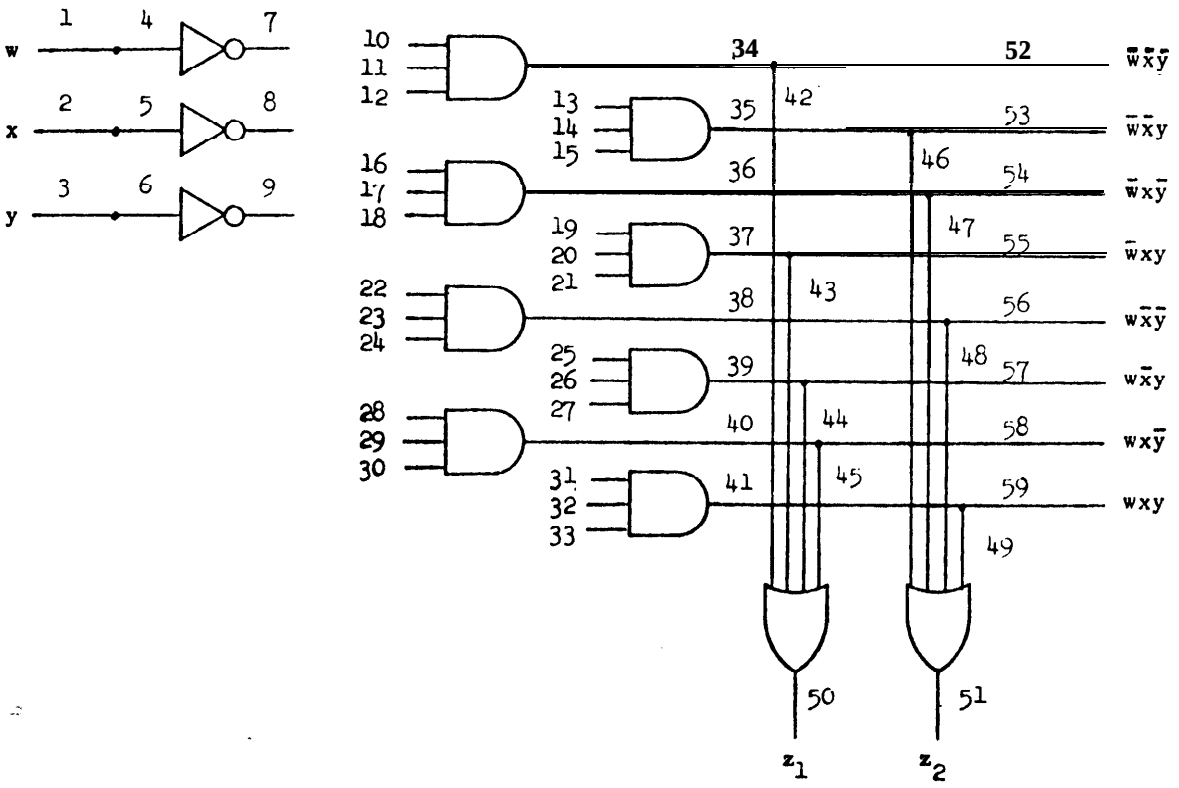


Fig. 1. - A one-out-of-eight decoder.

Table 1. - Conditional response probabilities of the single stuck-at faults in the network of Fig. 1 (equally-likely inputs).

Lines stuck-at 1	c_i	d_i	e_i	Lines stuck-at 0
1-3	.500	0	.500	1-3
52-59	.125	0	.875	
	.875	0	.125	52-59
4-9,42-51	.500	.500	0	4-9,50-51
10-33	.875	.125	0	10-49
34-41	.125	.500	.375	

Table 2 defines the dynamic-checking properties of a network with respect to a given fault F_i in terms of the values assumed by the conditional response probabilities of F_i.

Table 2. - The dynamic checking properties of a network.

untested	$d_i=0$	
self-testing	$d_i\neq0$	
secure		$e_i=0$
insecure		$e_i\neq0$
self- checking	$d_i\neq0$	$e_i=0$

The definitions in Table 2 are applicable to fault sets as well as individual faults. A network is self-testing for a fault set $\mathcal{F}$ for example, if it is self-testing for every member of $\mathcal{F}$. The network in Fig. 1 is self-testing for the set of single faults affecting lines 4-51, and self-checking for the set of stuck-at 0 faults affecting lines 4-51.

A fault in an imperfect-self-checking network need not cause an immediate detection or error response. The first error may precede or follow the first detection (Fig. 2).

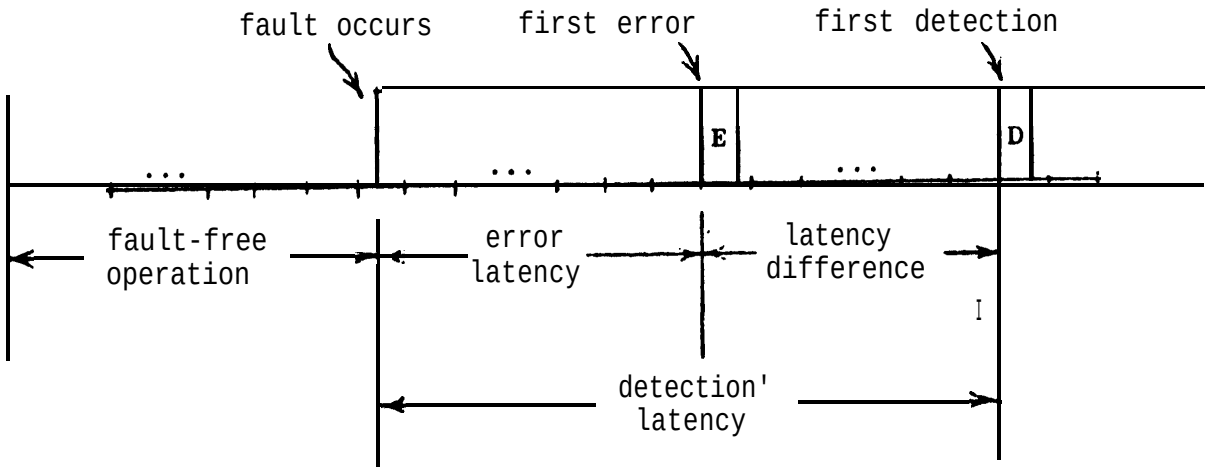


Fig. 2. - Latency in a self-testing network.

Definition. The error latency EL_i (detection latency DL_i) of a fault F_i is the number of input vectors applied to a network while F_i is active until the first error (detection) response due to F_i is observed.

The mean error (and detection) latency can be extremely large in some cases [7,8].

Definition. The latency difference LD. of a fault F_i is the difference between the detection latency DL_i and the error latency EL_i of F_i ($LD_i \equiv DL_i - EL_i$).

The latency difference is positive (negative) when the first erroroccursbefore (after) the first detection. The latency difference is never zero. The minimum rollback distance required for successful data restoration after a first time fault-detection is equal to $\max(LD_i, 0)$.

The latency difference is equal to positive n when the first error occurs, then n-l non-detections, and then the first detection. Any number of correct responses may precede the first error. The probability that the latency difference is equal to n is,

$$\Pr[LD_i = n] = (1+c_i+c_i^2+c_i^3+\ldots)(e_i(1-d_i)^{n-1}d_i)$$
$$= (d_ie_i/(e_i+d_i))(1-d_i)^{n-1};\ n>0 \qquad (2.2)$$

$$\Pr[LD_i = -n] = (1+c_i+c_i^2+c_i^3+\ldots)(d_i(1-e)^{n-1}e_i)$$
$$= (d_ie_i/(e_i+d_i))(1-e_i)^{n-1};\ n>0. \qquad (2.3)$$

The probability that the latency difference is greater than n is,

$$\Pr[LD_i > n] = \sum_{j=n+1}^{\infty} \frac{d_ie_i}{d_i+e_i}(1-d_i)^{j-1} = \frac{e_i}{d_i+e_i}(1-d_i)^n;\ n \geq 0. \qquad (2.4)$$

Finally, the probability distribution function for the latency difference is,

$$Pr[LD_i \leq n] = 1 - Pr[LD_i > n] = 1 - \frac{e_i}{e_i+d_i}(1-d_i)^n \; ; \; n > 0. \quad (2.5)$$

When the latency difference is less than or equal to n, the first error occurs after the first detection or at most n inputs before the first detection. Assuming that a program rollback restores data integrity, a successful data restoration occurs when the rollback is at least to the point of the first error. Equation (2.5) describes the probability of successful data restoration when the rollback distance from the point of detection of F_i is n inputs.

When n is zero, equations (2.4,2.5) have the following degenerate forms,

$$Pr[LD_i > 0] = Pr[\text{first E before first D}] = e_i/(e_i+d_i) \quad (2.6)$$

$$Pr[LD_i < 0] = Pr[\text{first D before first E}] = d_i/(e_i+d_i). \quad (2.7)$$

Equation (2.6) is implicitly used for the specific analysis of a checked decoder in [11].

Definition. The rollback interval $rb(x)_i$ of a fault F_i in a self-testing network is the minimum rollback distance (from the point of detection of F_i) necessary to have probability of successful data restoration at least equal to x.

If x is less than or equal to Pr[1st D before 1st E] then the rollback interval is zero. If x is greater than Pr[1st D before 1st E] then the rollback interval is a positive integer. A positive rollback interval is obtained by setting the left side of equation (2.5) to x and solving for n.

$$rb(x)_i = \max\left(\left\lceil \frac{\log(1-x)+\log(1+(d_i/e_i))}{\log(1-d_i)} \right\rceil , 0\right) \qquad (2.8)$$

Note that the rollback interval of a fault F_i is only defined when the network is self-testing ($d_i \neq 0$) for F_i. If the network is also secure ($e_i=0$) for F_i, the rollback interval is always zero.

Example 2. For the fault F_i=34 stuck-at 1 in Fig. 1, c_i=.125, d_i=.500, and e_i=.375. The probability distribution function of the latency difference is plotted in Fig. 3. The rollback interval $rb(.99)_i$ is 6 inputs; the probability of successful data restoration given the detection of F_i, is at least .99 when the rollback distance is 6 inputs or more.

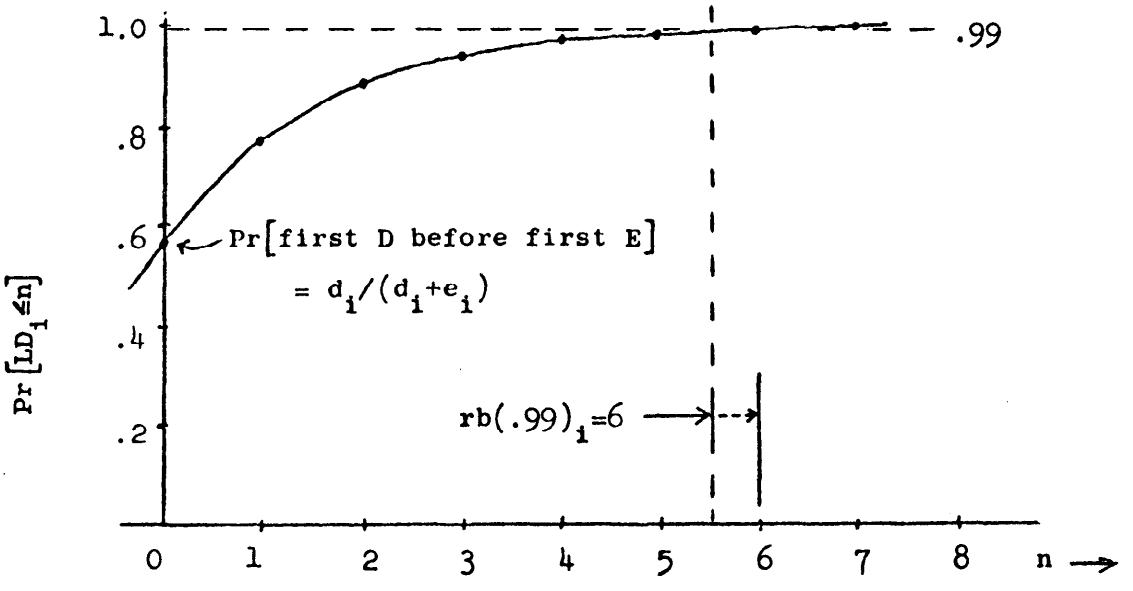


Fig. 3. - The probability distribution function of the latency difference of a fault F_i.

Theorem 1. If p_{min} is the probability of the least-likely input vector in the normal set $\mathfrak{N}$ of input vectors, then the upper-bound on the roll-back interval for any fault in an imperfect-self-checking network is denoted by $rb(x)_{ub}$ and is,

$$rb(x)_{ub} = \max\left(\left\lceil \frac{\log(1-x)}{\log(1-p_{min})} - 1 \right\rceil , 0\right) .$$

Proof. Equation (2.8) indicates that the rollback interval has maximum value when d_i has minimum value and e_i has maximum value. The minimum possible value of d_i is p_{min} while the maximum possible value of e_i is $1-P_{min}$. This condition occurs when the least-likely input vector causes a detection while the remaining input vectors cause an error.

When each input vector in the normal set $\mathfrak{N}$ is equally-likely then $P_{min} = 1/N$ where N is the cardinality of $\mathfrak{N}$. Table 3 lists the upper-bound rollback interval for this case.

Table 3. - Upper-bound rollback intervals for N equally-likely input vectors.

N	$rb(.95)_{ub}$	rb(	N	$rb(.95)_{ub}$	rb(
6	16	25	16	46	71
7	19	29	32	94	145
8	22	34	64	190	292
9	25	39	128	381	587
10	28	43	256	765	1176
.			512	1532	2355
.			1024	3066	4713
.					

One recovery philosophy is to set the rollback distance equal to the upper-bound rollback interval $rb(x)_{ub}$. Then the probability of successful data restoration is at least equal to x for any fault that may occur. This "upper-bound" philosophy uses a simple analysis that leads to a rough approximation (i.e. the upper-bound) of the rollback distance actually required.

Two other recovery philosophies use progressively finer approximations to the required rollback distance, but at a progressively greater cost of analysis. The increased accuracy generally allows the recovery philosophy to set a shorter rollback distance while maintaining a given probability of successful data restoration. A "worst-case" philosophy is discussed below and a "fault-set" philosophy is discussed in the following section.

In the set of reasonably likely faults in a network, the fault with the greatest rollback interval is the worst-case fault. The "worst-case" recovery philosophy sets the rollback distance equal to the rollback interval $rb(x)_{wc}$ of the worst-case fault. The probability of successful data restoration is at least equal to x for any fault that occurs.

The worst-case fault is discovered in two steps. The first step is to form a list L1 of reasonably likely faults with their corresponding conditional response probabilities. If the set of reasonably likely faults includes all single stuck-at faults, then only one member of each fault equivalence class [12,13] need be listed, If all multiple stuck-at faults are to be considered, then the enormous number of faults to be listed may be significantly reduced by a method discussed in the appendix to [8].

In step 2, the worst-case fault is chosen from the list L1. Unfortunately, the choice of the worst-case fault depends on the probability x of successful recovery desired, Example 3 illustrates the problem.

Example 3. Let L1 be the list of faults F_1, F_2, and F_3 below.

F_1: c_1 = ,930 d_1 = ,020 e_1 = .050

F_2: c_2 = ,985 d_2 = .010 e_2 = .005

F_3: c_3 = .994 d_3 = .005 e_3 = ,001

The probability distribution 'function of the latency difference is plotted in Fig. 4 for F_1, F_2, and F_3. If x1.286 then the rollback interval is 0 for all three faults and there is no worst-case fault. If .286<x≤.843 then the worst-case fault is F_1. If .843<x≤.916 then the worst-case fault is F_2, and if x>.916 the worst-case fault is F_3.

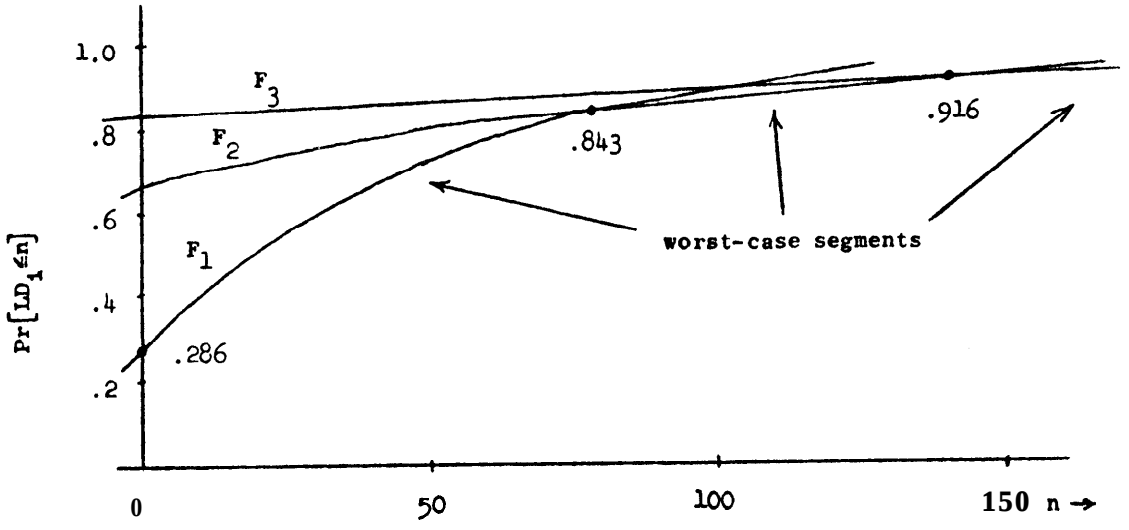


Fig. 4. - The probability distribution function8 of three faults, each of which is a worst-case fault in turn.

The following algorithm forms a greatly reduced list L2 of faults from which the worst-case fault is chosen.

Algorithm 1.

1. Initially all faults in list **L1** are unmarked and list L2 is empty. Mark those faults in **L1** whose $d_i=0$ or whose $e_i=0$.
2. Set k equal to the minimum of the values d_i/e_i corresponding to the unmarked faults in list **L1.**
3. From the faults with d_i/e_i equal to k, choose the fault F_j with minimum valued d_j. Set $g=d_j$ and add F_j to list L2.
4. In list **L1,** mark all the faults with d_i/e_i equal to k and all faults with di $\geq$ g. If an unmarked fault remains in **L1,** go to step 2. Otherwise exit.

Theorem 2. If a worst-case fault for a given probability x of successful data restoration exists, then it is in the list L2 generated by algorithm 1.

Proof. Step 1 eliminates faults for which the network is untested or secure. The rollback interval is undefined in the former case and zero in the latter case. A fault is discarded when it is not added to list L2 in step 3 and then marked in step 4. The probability distribution function $\Pr[LD_i \geq n]$ of a discarded fault is always greater than or equal to the **probability distribution** function of the fault chosen for list L2; it is initially greater or equal at **n=0** and rises toward 1 at a greater or equal rate as n increases. Hence a discarded fault can never have a rollback interval greater than the fault chosen for list L2.

Algorithm 1 is surprisingly effective in reducing the search for the worst-case fault. In section 4 a list L1 of 156 faults is reduced to a list L2 of 2 faults. The worst-case fault is chosen from list L2 by finding the fault with the greatest rollback interval for a given probability x of successful data restoration.

THE ROLLBACK INTERVAL OF A FAULT SET IN AN IMPERFECT-SELF-CHECKING NETWORK

The worst-case recovery philosophy fails to assess the beneficial effects of the faults that are not worst-case but may be just as likely to occur. The following fault-set latency measure computes the average effect of all the faults in a fault set.

Definition. The error latency $EL_{\mathcal{F}}$ (detection latency $DL_{\mathcal{F}}$) of a fault set $\mathcal{F}$ is the number of input vectors applied to a network while one fault in $\mathcal{F}$ is constantly active, until the first error (detection) due to the fault is observed. Inherent in the definition and use of the fault-set error (detection) latency is an assumption that the one particular fault that becomes active is chosen from $\mathcal{F}$ according to some conditional probability distribution.

Each fault in $\mathcal{F}$ has a conditional probability of being the one fault that occurs, given that some fault in $\mathcal{F}$ must occur. The conditional probabilities must sum to 1.

$$\sum_{i:F_i \in \mathcal{F}} Pr[F_i | \mathcal{F}] = 1 \qquad (3.1)$$

The error latency of a fault set is discussed in detail in 18,141.

Definition. The latency difference $LD_{\mathcal{F}}$ of a fault set $\mathcal{F}$ is the difference between $DL_{\mathcal{F}}$ and $EL_{\mathcal{F}}$, where the network is self-testing for every fault in $\mathcal{F}$ ($LD_{\mathcal{F}} \equiv DL_{\mathcal{F}} - EL_{\mathcal{F}}$).

The probability that the fault-set latency difference is equal to n is obtained by summing the probabilities of the disjoint union of subevents indicated below,

$$\Pr[LD_{\mathcal{F}}=n] = \Pr\Big[\bigcup_{i;F_i\in\mathcal{F}} \{LD_{\mathcal{F}}=n,\ F_i\}\Big] =$$

$$\sum_{i:F_i\in\mathcal{F}} \Pr[LD_{\mathcal{F}}=n,\ F_i] = \sum_{i:F_i\in\mathcal{F}} \Pr[LD_i=n]\Pr[F_i|\mathcal{F}] \qquad (3.2)$$

The probability distribution function of the fault-set latency difference $LD_{\mathcal{F}}$ is an average of the probability distribution functions of the latency difference LD_i for each fault F_i in $\mathcal{F}$,

$$\Pr[LD_{\mathcal{F}}\le n] = 1 - \sum_{j=n+1}^{\infty} \Pr[LD_{\mathcal{F}}=j] =$$

$$1 - \sum_{i:F_i\in\mathcal{F}} \left(\sum_{j=n+1}^{\infty} \Pr[LD_i=n]\Pr[F_i|\mathcal{F}] \right) =$$

$$\sum_{i:F_i\in\mathcal{F}} \Pr[LD_i\le n]\Pr[F_i|\mathcal{F}] \qquad (3.3)$$

Definition. The rollback interval $rb(x)_{\mathcal{F}}$ of a fault set $\mathcal{F}$ in a self-testing network is the minimum rollback distance necessary to have probability at least equal to x of successful data restoration from an undiagnosed but detected fault F_i belonging to $\mathcal{F}$.

The fault-set rollback interval $rb(x)_{\mathcal{F}}$ is the minimum value of n that makes the left side of equation (3.3) at least equal to x.

When $\mathcal{F}$ is the set of all reasonably likely faults, the probability x specified for $rb(x)_{\mathcal{F}}$ is the overall probability of successful data restoration. The coverage factor [15,16] of a system is the probability of system recovery given a fault, Since successful data restoration is a prerequisite of successful recovery, the probability x is an important

factor of the coverage factor. To say the least, the coverage factor cannot be greater than x. Because the mean time to system failure is extremely sensitive to deviations in the value of the coverage factor from 1 [15,16], it is critical that the rollback distance be great enough to insure a value of x close to 1.

Example 4. The network in Fig. 1 is self-testing for the single faults involving lines 4-51. Table 4 lists the rollback interval for this fault set when each fault in $\mathcal{F}$ is equally-likely.

	rb(.90)	rb(.99)
upper-bound	18	34
worst-case	3	6
fault-set $\mathcal{F}$	0	2

$\mathcal{F}$ is the set of single faults on lines 4-51.

Table 4. - Rollback intervals for the network In Fig. 1 (equally-likely inputs).

THE ROLLBACK INTERVAL OF AN IMPERFECT-SELF-CHECKING ARITHMETIC AND LOGIC UNIT

In this section we determine the rollback interval of a moderately complex example, the arithmetic and logic unit (ALU) contained in the imperfect-self-checking network in Fig. 5. The network in Fig. 5 exemplifies the imperfect-self-checking design techniques used in the Stanford SDC-16 computer [9,17]. We choose to analyze the ALU since it is the most "insecure" component of the otherwise self-checking SDC-16.

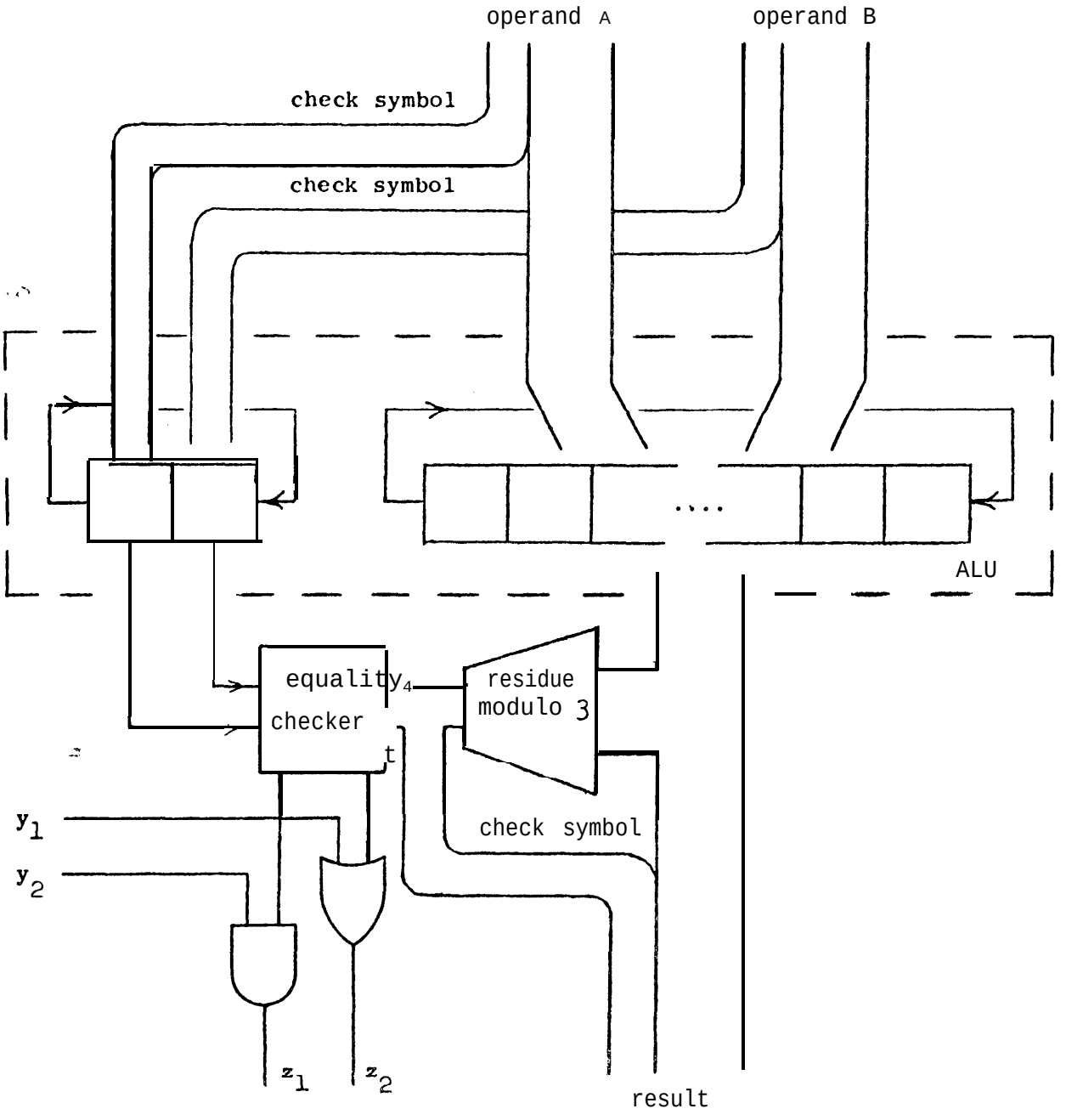


Fig. 5. - A simplified diagram of an imperfect-self-checking ALU network.

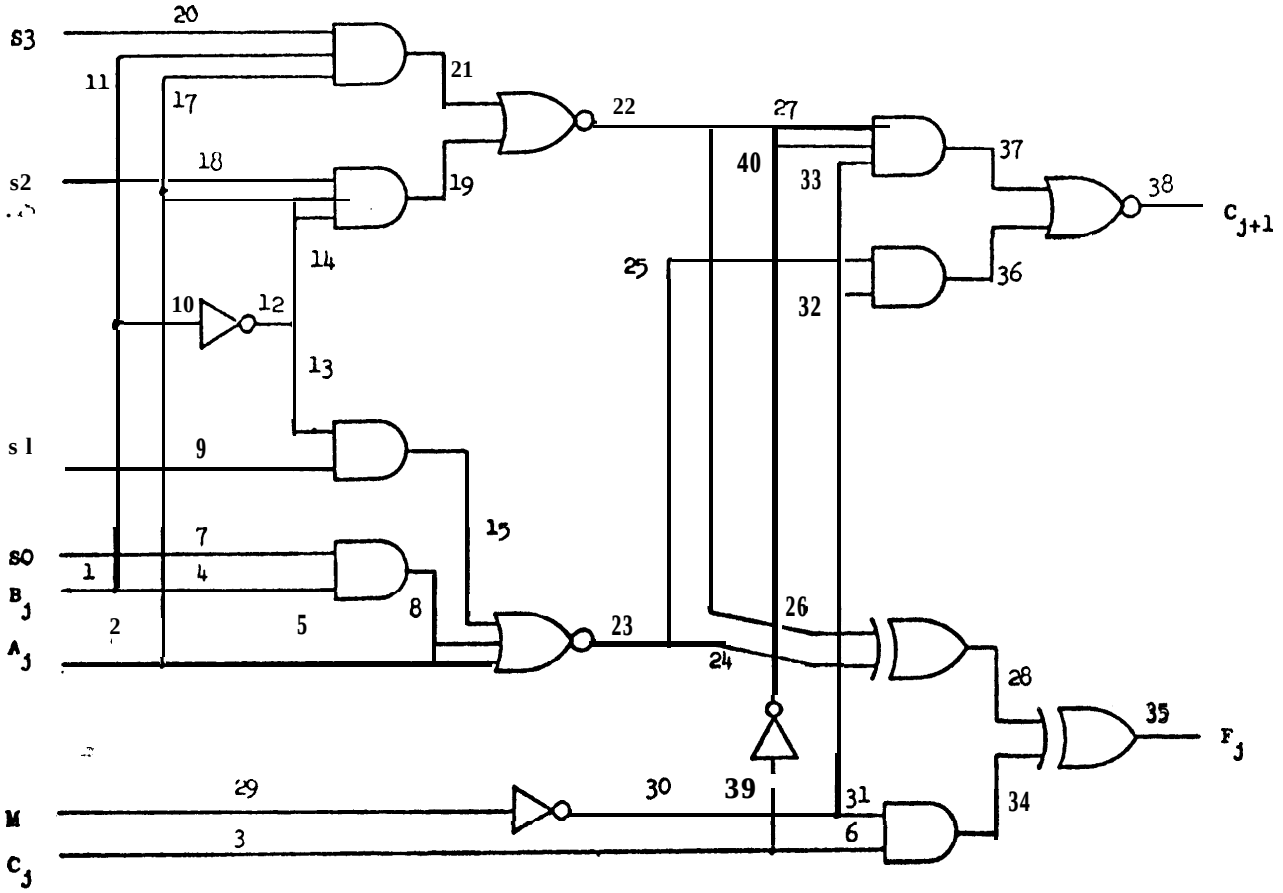
The network in Fig. 5 contains a bit-sliced (Fig.6), ripple-carry ALU composed of separate check symbol and data part sections, and a checker/re-encoder. Operands are encoded in an error-detecting code preserved by arithmetic operations, but not by logic operations.* The checker is enabled ($y_1 y_2$ = <0 1>) during code preserving operations, so that a detection is signaled ($z_i z_2$ = <0 0> or <1 1>) if the preserved check symbol does not agree with the check symbol generated from the output of the ALU data section. The checker is disabled ($y_1 y_2$= <1 0>) during non-preserving operations, allowing errors produced by the ALU data section to escape detection and be re-encoded. In this case an error occurs.

The code is a low-cost residue code [19] with group length a=2. A codeword has a 16-bit data part and a 2-bit check symbol. The check symbol is equal to the data part modulo $2^a-1=3$, with check symbol <1 1> recognized equivalent to <0 0> . Both the check symbol and data part sections of the ALU have end-around-carry so that arithmetic operations are modulo 2^2-1 and modulo 2^{16}-1 respectively (1's complement).

The effect of an arithmetic *error* on a correct ALU output V is to change it to $V_{plus} R$ where in coding terminology, R is the error value. This code detects all error values of the forms $\pm 2^j$ and $\pm 2^{j+i} \mp 2^j$ [17, 20). An error value of the form $\pm 2^{j+i} \pm 2^j$ is not detected.

For example, line 28 stuck-at- 1 in a data section bit-slice affects

*There is no error-detecting code short of duplication for logic operations [18]. The preserving arithmetic operations are : PLUS, MINUS, LR, $\bar{B}$, B, $\bar{A}$, A, all l's, and all 0's. The logic operations are: AB, A+B, A⊕B.



	S3 - S0	M	mode	operation freq.
A⊕B	1 0 0 1	1	insecure	.02
A+B	0 0 0 1	1	insecure	.04
AB	0 1 0 0	1	insecure	.04
$\overline{B}$	1 0 1 0	1	secure	.03
B	0 1 0 1	1	secure	.22
$\overline{A}$	1 1 1 1	1	secure	.03
A	0 0 0 0	1	secure	.22
1	0 0 1 1	1	secure	.03
0	1 1 0 0	1	secure	.06
LR	1 1 0 0	0	secure *#	.01
Plus	1 0 0 1	0	secure *	.20
Minus	0 1 1 0	0	secure	.10

*except for line 23 stuck-at 1

#except for line 5 stuck-at 0

Fig. 6. - An ALU bit-slice and operation frequcncics.

only the output F_j, so it can only cause an error value of $\pm 2^j$. This fault can cause a correct response or a detection during an add. Line 23 stuck-at 1 in a data section bit-slice affects both the output F_j and the carry-out C_{j+i}. It causes the error values -2^j, $-2^{j+1}+2^j$, and $-2^{j+i}-2^j$, depending on the values of the input lines A_j, B_j, and c_j. This fault may cause a correct response, a detection, or an error during an add.*

The ALU is self-testing for the set of single faults [17]**, insecure for logic operations, and except for lines 23 and 5 stuck-at-l, secure for arithmetic operations.

Lemma 1 simplifies the analysis of the ALU,

Lemma 1. Let $c_i(Y_k)$, $d_i(Y_k)$, and $e_i(Y_k)$ be the conditional response probability components of a fault F_i in a network where input subvector Y is fixed at Y_k. If input subvector Y is independent, then $c_i = \sum_k c_i(Y_k) \Pr[Y_k]$. The same is true for the response probabilities d_i and e_i.

Assuming the control input vector is independent of the operand vectors, lemma 1 permits the calculation of the conditional response probabilties by determining the conditional response probability components for each operation, then taking an average according to the probability of each operation.

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*Line 5 stuck-at1 is the only other single fault in the ALU for which the network of Fig. 5 is not secure during all arithmetic operations.

**In the analysis to follow, the conditional response probabilities of of the lines 32 and 33 stuck-at 1 (Fig. 6) are $c_i=1.0$, $d_i=0$, $e_i=0$, so they are excluded form the set of single faults considered.

The analysis strategy is as follows, Simulate each single stuck-at-fault in the bit slice of Fig. 6, Associate the input lines S0-S3, M with a control subvector Y, and the operand lines A_j, B_j and carry-in line C_j with an operand subvector. For each fault F_i, and for each operation specified by a control subvector Y_k, determine the error value in the output of the ALU caused by the application of each of the possible operand subvectors. Analyze the error values so that each operand subvectorcan be classified as eliciting either an error, a detection, or a correct response, The probabilities of these operand subvectors are summed to obtain the conditional response probability components $c_i(Y_k)$, $d_i(Y_k)$, and $e_i(Y_k)$. The conditional response probabilities c_i, d_i, and e_i are calculated using lemma 1.

An end-around-carry adder as shown in Fig. 5 has a somewhat sequential nature when the input lines A_j and B_j to every bit-slice are carry-propagating inputs (A_iB_i = <0 1> or <1 0>) [21]. This situation occurs only when a number and its l's complement are added. The sum may be plus or minus zero (all 0's or all l's) depending if the carry lines are all 1 or all 0, Both states of the carry lines are stable, and the final state reached depends on the previous condition of the carry lines and the relative propagation delays of the bit-slices, Since both ±0 are recognized by the 1's complement number system employed here, this sequential nature is no theoretical problem. The duration of the race before a stable state is reached however, can cause timing problems, so some method to drive the carry lines to a stable state is desirable.

The probabilities of the operand subvectors applied to a bit-slice

table 5. - The probability of applying the operand subvcctor A_j, B_j, C_j to the j^{th} ALU bit-slice during the indicated operation.

$A_jB_jC_j$	A⊕B	A+B	AB	$\overline{B}$,B	$\overline{A}$,A	0,1	LR	Plus	Minus	
000	0	0	0	0	0	0	0	p+ =.125004	p+ =.125004	DATA SECTION
001	$(1-p_a)(1-p_b)$ =.25	$(1-p_a)(1-p_b)$ =.25	$(1-p_a)(1-p_b)$ =.25	0	0	0	0	p- =.124996	p- =.124996	
010	0	0	0	0	0	0	$(1-p_a)(1-p_a)$ =.25	p+ =.125004	p- =.124996	
011	$(1-p_a)p_b$ =.25	$(1-p_a)p_b$ =.25	$(1-p_a)p_b$ =.25	0	$1-p_a$ =.5	0	$(1-p_a)p_a$ =.25	p- =.124996	p+ =.125004	
100	0	0	0	0	0	0	0	p+ =.125004	p- =.124996	
101	$p_a(1-p_b)$ =.25	$p_a(1-p_b)$ =.25	$p_a(1-p_b)$ =.25	$1-p_b$ =.5	0	0	0	p- =.124996	p- =.124996	
110	0	0	0	0	0	0	$p_a(1-p_a)$ =.25	p- =.124996	p+ =.125004	
111	p_ap_b =.25	p_ap_b =.25	p_ap_b =.25	p_b =.5	p_a =.5	1	p_ap_a =.25	p+ =.125004	p- =.124996	
000				0	0	0	0	$p_0p_0+2p_0p_{12}$ =.138891	$p_0p_0+p_0p_{12}+p_{12}p_{12}$ =.194443	CHECK SECTION
001				0	0	0	0 $p_{12}p_{12}$	=.111108	p_0p_{12} =.055556	
010				0	0	0	p_0 =.166672	$p_0p_{12}+p_0p_3+p_{12}p_{12}$ =.194443	$p_0p_{12}+p_0p_3+p_{12}p_3$ =.138891	
011				0	$p_{12}+p_0$ =.5		p_{12} =.333328	$p_{12}p_3$ =.055556	$p_{12}p_{12}$ =.111108	
100				0	0	0	0	$p_0p_{12}+p_0p_3+p_{12}p_{12}$ 1.194443.	$p_{12}p_{12}$ =.111108	
101				$p_{12}+p_0$ =.5	0	0	0	$p_{12}p_3$ =.055556	$p_0p_{12}+p_0p_3+p_{12}p_3$ =.138891	
110				0	0	0	p_{12} =.333328	$p_{12}p_{12}$ =.111108	$p_{12}p_{12}+p_{12}p_3+p_3p_3$ =.194443	
111				$p_{12}+p_3$ =.5	$p_{12}+p_3$ =.5	1	p_0 ..166672	$2p_{12}p_3+p_3p_3$ =.138841	$p_{12}p_3$ =.055556	
comments	C_j=1*	C_j=1	C_j=1	C_j=1 A_j=1#	C_j=1 B_j=1	C_j=1 A_j=1, B_j=1	B_j=1	C_j is carry	C_j is $\overline{\text{borrow}}$	

For all single faults in the ALU check section: c_i=1.0, d_i=0, e_i=0 (because checker is disabled)

p_a = Pr[A_j=1] (data section) = .5

p_b = Pr[B_j=1] (data section) = .5

p_0 = Pr[operand check symbol =⟨00⟩] = $(1/6)(1 + 1/2^{15})$ = .166672

p_{12} = Pr[operand check symbol =⟨01⟩] = Pr[operand chock symbol =⟨10⟩] = $(1/3)(1 - 1/2^{16})$ = .333328

p_3 = Pr[operend check symbol =⟨11⟩] = $(1/6)(1 + 1/2^{15})$ = .166672

p+ = 4 8 + $1/2^{18}$

p- = 1/8 - $1/2^{18}$

*M=1 implies C_j=1

#Assume bus line A_3 □□ B_j is 1 when an operand is absent.

are listed in Table 5. Table 5 assumes that the carry lines are initially driven to 0 for arithmetic operations [21], that the operand lines A_j and B_j are 1 in the absence of an operand, that the signal probabilities [22] of lines A_j and B_j in the data section are independent and equal to .5 for an operand, and that the proportions of the check sumbols <0 0> and <1 1> (both are recognized as 0 modulo 3 [17]) are equal.

Since the operand subvector probabilities are the same for each bit-slice in the check section and for each bit-slice in the data section of the ALU, the conditional response probabilities of the same faults in different bit-slices of those sections are the same. We need only examine one bit slice from each section.

When only faults restricted to a bit-slice are considered, the upper-bound rollback interval for the ALU in Fig. 5 is equal to the upper-bound rollback interval of the bit-slice in Fig. 6. The minimum input-vector probability p_{min} for a bit-slice is the minimum of the products of the control and operand subvector probabilities. Using Table 5 and the example operation mix listed in Fig. 6, $p_{min} = (.01)(.166672) = .001667$ and theorem 1 gives the upper-bound rollback interval.

A worst-case single fault for the ALU is among the 156 single faults examined in the check and data section bit-slices (78 faults each). Using algorithm 1, the list L1 of 156 faults is reduced to a list L2 of 2 faults. The worst-case fault. for both entries in Table 6 is 16 stuck-at 1 in the data section bit-slice. Table 6 lists the upper-bound and worst-case rollback intervals for the ALU. Table 6 also lists the fault-set rollback interval for the set of (78)(18)=1404 single faults in the ALU.

Table 6. - Rollback intervals for the ALU In Fig. 5.

	rb(.90)	rb(.99)
upper- bound	1380	2760
worst-case	42	133
single-fault set	0	J-6

Pr[1st D before 1st E] = .936

Table 6 clearly demonstrates the smaller values obtained for the approximation to the required rollback distance by the increasingly accurate and costly analysis methods.

In the example operation mix in Fig. 6, the probabilities of the logic operations (AND, OR,and XOR) sum to .10. Increasing the relative proportion of these insecure logic operations should increase the required rollback distance. Fig. 7 plots the rollbackinterval of the ALU against the probability of a logic operation. Relative operation probabilities within the groups of logic and arithmetic operations remain the same as in Fig. 6. The rollback interval is roughly linear with the probability of a logic operation.

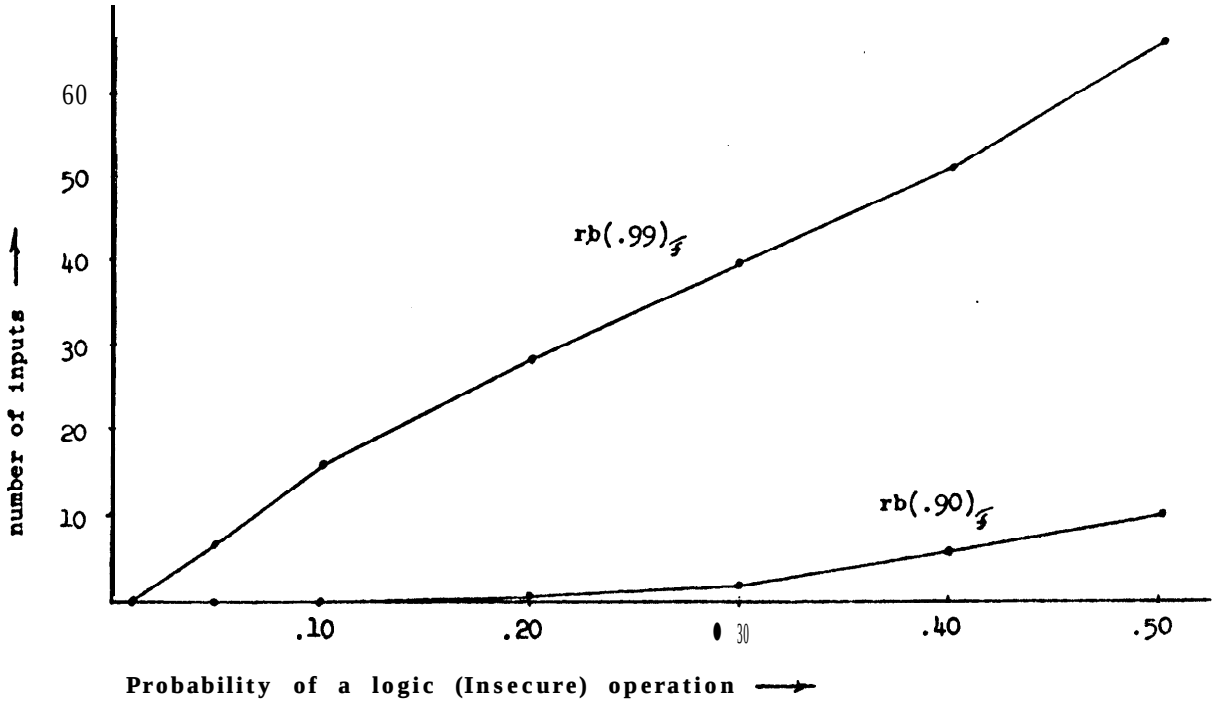

Fig. 7. - The single-fault set latency interval for the ALU in Fig. 5 as a function of the probability of a logic operation.

SUMMARY

The effectiveness of a self-testing network is best measured by the rollback distance required when a fault is detected. The upper-bound, worst-case, and fault-set rollback intervals are three increasingly accurate measures of this distance.

The analysis directly relates the rollback distance to the probability of successful data restoration. Since the probability of successful data restoration is an upper-bound on the coverage factor, it follows that the overall system reliability strongly depends on the rollback distance. The analysis offers a way to choose the appropriate rollback distance.

An inexpensive, imperfect-serf-checking ALU was shown to require a rollback distance of only 16 inputs to have probability .99 of successful data restoration when a single fault is detected.

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