Introduction to Automata and Complexity Theory

Winter Quarter 2003

Homework 2 (01/15/03)

Due: Wednesday 01/29 in class

- Solving all of the easy problems can substitute for solving a single moderate problem.
- Each problem must be submitted on a separate sheet.
- Please write your name and SUnet ID at the beginning of each problem. We will use your SUnet ID to communicate grades to you.

Easy Problems

Problem 1 Prove that the following languages are not regular.

(a) $\{ 0^i 1^j \mid i > j \}$

(b) $\{ 0^m 1 0^n 1 0^{m+n} \mid m, n > 0 \}$

Problem 2

(a) Show that the class of context-free languages is closed under union, concatenation, and star.

Hint: Very easy to show using context-free grammars.

(b) Using (a), show that every regular language is context-free, by showing how to convert a regular expression directly to an equivalent context-free grammar.

Problem 3 Let $G = (V, \Sigma, R, S)$ be the following grammar:

$V = \{S, T, U\}$, $\Sigma = \{0, \#\}$

R:

$S \to TT \mid U$
$T \to 0T \mid T0 \mid \#$
$U \to 0U00 \mid \#$

(a) Describe $L(G)$ in English.

(b) Prove that $L(G)$ is not regular.

Problem 4 Begin with the grammar:

$$\begin{aligned} S &\rightarrow aAa \mid bBb \mid \epsilon \\ A &\rightarrow C \mid a \\ B &\rightarrow C \mid b \\ C &\rightarrow CDE \mid \epsilon \\ D &\rightarrow A \mid B \mid ab \end{aligned}$$

(a) Eliminate ϵ-productions.

(b) Eliminate unit productions.

(c) Eliminate useless symbols.

(d) Put the grammar into Chomsky normal form.

Problem 5 Let $L = \{ w \in \{0,1\}^* \mid w = w^R \}$, where $\Sigma = \{0,1\}$.

(a) Construct a context-free grammar for L.

(b) Construct a pushdown automata for L.

Moderate Problems

Problem 1 Prove that for any DFA $M = (Q, \Sigma, \delta, q_0, F)$, M accepts an infinite language if and only if M accepts some string of length greater than or equal to $|Q|$ and less than $2|Q|$.

Problem 2

(a) Give a CFG for $L_1 = \{ a^i b^j c^k \mid i = j \text{ or } j = k \}$.

(b) Give a PDA for L_1. Show clearly the transition diagram. Also, provide a verbal description of its operation. Do not use any extra stack symbols apart from $\{a, b, c, Z_0\}$.

Problem 3 Let $G = (V, \Sigma, R, S)$ be the following grammar:

$V = \{S\}$, $\Sigma = \{a, b\}$
R: $S \rightarrow aSbS \mid bSaS \mid \epsilon$

Prove that $L(G) = \{ w \mid w \text{ has same number of } a\text{'s as } b\text{'s} \}$.

Problem 4 Design PDAs to accept each of the following languages:

(a) $\{ a^i b^j c^k \mid i \neq j \text{ or } j \neq k \}$

(b) $\{ w \mid w \text{ has twice as many 1's as 0's} \}$

Problem 5 Given $\Sigma = \{0, 1, \#\}$, let language L be defined as

$$L = \{\, x\#y \mid x, y \in \{0,1\}^*,\ x \neq y \,\}$$

Show that L is a context-free language.

Extra Credit Problem

If a context-free grammar G has no "self-embedding" nonterminal, prove that $L(G)$ is regular. A self-embedding nonterminal is a nonterminal A for which there is a derivation $A \overset{*}{\Rightarrow} uAv$, where u and v are both non-empty strings of terminals and nonterminals.