Introduction to Automata and Complexity Theory

Winter Quarter 2003

Homework 3
Due Wed Feb 5, in class

Note: All these problems except easy 5 are taken from the textbook.
Please write your **Leland login ID** on your submission.
You are allowed to take at most 1 late day for this homework. (We would like to post solutions by Feb 6. The midterm is on Feb 7.)

Easy Problems

Problem 1. Give context-free grammars generating the following languages:

(a) $\{ w\#x \mid w^R \text{ is a substring of } x, \text{ for } w,x \in \{0, 1\}^* \}$

(b) $\{ x_1\#x_2\#\ldots\#x_k \mid k \geq 1, \text{ each } x_i \in \{0,1\}^*, \text{ and for some } i \neq j, x_i = x_j^R \}$

Problem 2a,b. Give PDAs for the languages of Problem 1.

Problem 3. Construct a Turing Machine for $\{ a^n b^n c^n \}$. Show the state diagram.

Problem 4. Show the sequence of configurations the machine enters on input "abcc", for the machine of Problem 3.

Problem 5. (*Good review for midterm*) For each of these languages, determine whether the language, or its complement, is (A) finite (B) regular (C) context-free (D) neither. No need to prove.

(a) $\{ 0^i\#1^i \mid i \geq 1 \}$
(b) $\{ xy \mid x \neq y \}$; $\Sigma = \{0,1\}$
(c) $\{ xy \mid |x| = |y| \text{ and } x \neq y \}$ $\Sigma = \{0,1\}$
(d) $\{ x\#x\#x \mid |x| \leq 10^{10} \}$ $\Sigma = \{0,1,\#\}$
(e) $\{ x\#x\#x \mid |x| \geq 10^{10} \}$ $\Sigma = \{0,1,\#\}$
(f) $\{ x\#y \mid x, y \text{ are binary numbers, and } x = y \pmod 5 \}$

Moderate Problems

Problem 1. Use the pumping lemma to show that the following languages are not context-free:

(a) $\{ 0^n1^n0^n1^n \mid n \geq 0 \}$
(b) $\{ w\#x \mid w \text{ is a substring of } x, \text{ where } w,x \in \{a,b\}^* \}$

(c) $\{ x_1\#x_2\#...\#x_k \mid k \geq 2$, each $x_i \in \{a,b\}^*$, and for some $i \neq j$, $x_i = x_j \}$

Problem 2.

(a) Give an example of a context free language whose complement is not context free. Prove that your example works.
(b) Give an example of a language that is not context free but that does satisfy the Pumping lemma. Prove that your example works.

Problem 3. Construct a Turing Machine for the language
$\{ w \in \{0,1\}^* \mid w$ contains twice as many 0s as 1s $\}$

(a) Just give the description
(b) Give the state diagram
(c) Give the sequence of configurations on input "011"

Problem 4. Show that the collection of decidable languages is closed under (a) Union, (b) Concatenation, (c) Star, (d) Complementation, and (e) Intersection.

Problem 5. Show that the collection of Turing-recognizable languages is closed under (a) Union, (b) Concatenation, (c) Star, and (d) Intersection.

Extra Credit Problem. Prove the following stronger form of the pumping lemma, wherein we require both pieces v and x to be nonempty when w is broken up:

If L is a context-free language, and $G = (V, \Sigma, R, S)$ is a CFG in CNF for L, then there is a number p where, if w is any string in L of length $|w| \geq p$, then w may be divided into five pieces $w = uvxyz$ such that:

1. For each $i \geq 0$, $uv^ixy^iz \in L$
2. $v \neq \varepsilon$ and $y \neq \varepsilon$
3. $|vxy| \leq k$

In your proof make sure to demonstrate the pumping length p that works for language L, in terms of grammar G (e.g., in terms of the sizes |V|, |R|, etc).

Extra Credit Problem – Due with Problem Set 4. Say that a write-once Turing Machine is a single-tape TM that can alter each tape square at most once (including the input portion of the tape). Show that this variant Turing machine model is equivalent to the ordinary Turing machine model.

Hint: As a first step consider the case whereby the Turing machine may alter each tape square at most twice. Use lots of tape.