cs154 Homework #4, Winter 2002-2003

Due date: Feb 19, 2003, in class.

Please write your **Leland login ID** for each problem you submit.

Easy Problems

(A) Consider the problem of testing whether a DFA and a regular expression are equivalent.

Express this problem as a language and show that it is decidable.

(B) Let $\mathcal{B}$ be the set of all infinite sequences over $\{0, 1\}$.

Show that $\mathcal{B}$ is uncountable, using a proof by diagonalization.

(C) Let $T = \{(i, j, k) \mid i \leq j \leq k \text{ and } i, j, k \in \mathcal{N}\}$. Show that T is countable.

(D) Show that EQ_{CFG} is undecidable.

(E) Show that EQ_{CFG} is the complement of a Turing recognizable language.

Problem 1 Let $A = \{\langle R, S\rangle \mid R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$.

Show that A is decidable.

Problem 2 Show that the problem of testing whether a CFG generates all strings in 1^* is decidable.

In other words, show that $\{\langle G\rangle \mid G \text{ is a CFG over } \{0,1\}^* \text{ and } 1^* \subseteq L(G)\}$ is a decidable language.

Problem 3 Let $E = \{\langle M\rangle \mid M \text{ is a DFA that accepts some string of the form } ww^R \text{ for } w \in \{0,1\}^*\}$.

Show that E is decidable.

Problem 4 Let $J = \{w \mid w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$.

Show that neither J nor $\overline{J}$ is Turing-recognizable.

Problem 5 Let $S = \{\langle M\rangle \mid M \text{ is a TM that accepts } w^{\mathcal{R}} \text{ whenever it accepts } w\}$.

Show that S is undecidable.

Extra Credit

Part (a) Say that a write-once TM is a single-tape TM that can alter each tape square at most once (including the input portion of the tape). Show that this variant TM model is equivalent to the ordinary TM model.

Hint: As a first step, consider the case whereby the TM may alter each tape square at most twice. Use lots of tape.

Part (b) (Rice's Theorem) Let P be any problem about Turing machines that satisfies the following two properties. As usual we express P as a language.

a. For any TMs M_1 and M_2, where $L(M_1) = L(M_2)$, we have $\langle M_1\rangle \in P$ iff $\langle M_2\rangle \in P$. In other words, the membership of a TM M in P depends only on the language of M.

b. There exist TMs M_1 and M_2, where $\langle M_1\rangle \in P$ and $\langle M_2\rangle \notin P$. In other words, P is non-trivial — it holds for some, but not all, TMs.

Show that P is undecidable.

Note: **Do not use Rice's Theorem to solve any of the moderate problems in this homework.**