

cs154 Homework #6, Winter 2002-2003

Due date: Wednesday, Mar 12, 2003, in class.

Please write your **Leland login ID** for each problem you submit.

Easy Problems

(A) The *Set Partition Problem* takes as input a set S of numbers. The question is whether the numbers can be partitioned into two sets A and $\bar{A} = S - A$ such that $\sum_{x \in A} x = \sum_{x \in \bar{A}} x$. Show that Set Partition is NP-Complete. *Hint: Reduce SUBSET-SUM.*

(B) Show that the *subgraph-isomorphism problem* is NP-complete: Given two graphs G_1 and G_2 , does G_1 contain a copy of G_2 as a subgraph? That is, can we find a subset of the nodes of G_1 that, together with the edges among them in G_1 , forms an exact copy of G_2 when we choose the correspondence between nodes of G_2 and nodes of the subgraph of G_1 properly? *Hint: Reduce CLIQUE.*

(C) Consider the *graph-isomorphism problem*: Given two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, is there a 1-1 mapping $\pi : V_1 \rightarrow V_2$ such that $(u, v) \in E_1$ iff $(\pi(u), \pi(v)) \in E_2$? (a) Is the graph-isomorphism problem reducible to subgraph-isomorphism problem? (b) Is the subgraph-isomorphism problem reducible to the graph-isomorphism problem? (c) Is the graph-isomorphism problem in NP?

(D) Given an integer $m \times n$ matrix A and an integer m -vector b , the *0-1 Integer Programming Problem* asks whether there is an integer n -vector x with elements in the set $\{0, 1\}$ such that $Ax \leq b$. Prove that 0-1 Integer Programming is NP-Complete. *Hint: Reduce 3SAT.*

(E) Let $DOUBLE - SAT = \{\langle \phi \rangle \mid \phi \text{ has at least two satisfying assignments}\}$. Show that $DOUBLE - SAT$ is NP-Complete. *Hint: Reduce 3SAT.*

Problem 1 (Sipser 7.23) *MAX - CUT*

Problem 2 (Sipser 7.24) Polynomial with integral root.

Problem 3 (Sipser 7.34) *3 - COLOR*

Problem 4 (a) (Sipser 7.21) *HALF - CLIQUE* **(b)** (Sipser 7.30) *MAX - CLIQUE*

Problem 5 (a) (Sipser 7.28) $P = NP$ and Factoring **(b)** (Sipser 7.29) $P = NP$ and Satisfiability

Extra Credit I (10 Points) (Sipser 7.37) $2CNF \in P$

Extra Credit II (10 Points) (Sipser 7.18) $PRIMES \in NP$.