

# Everything Else About Data Flow Analysis

Flow- and Context-Sensitivity  
Logical Representation  
Pointer Analysis  
Interprocedural Analysis

# Three Levels of Sensitivity

- ◆ In DFA so far, we have cared about where in the program we are.
  - ◆ Called *flow-sensitivity*.
- ◆ But we didn't care how we got there.
  - ◆ Called *context-sensitivity*.
- ◆ We could even care about neither.
  - ◆ **Example**: where could **x** ever be defined in this program?

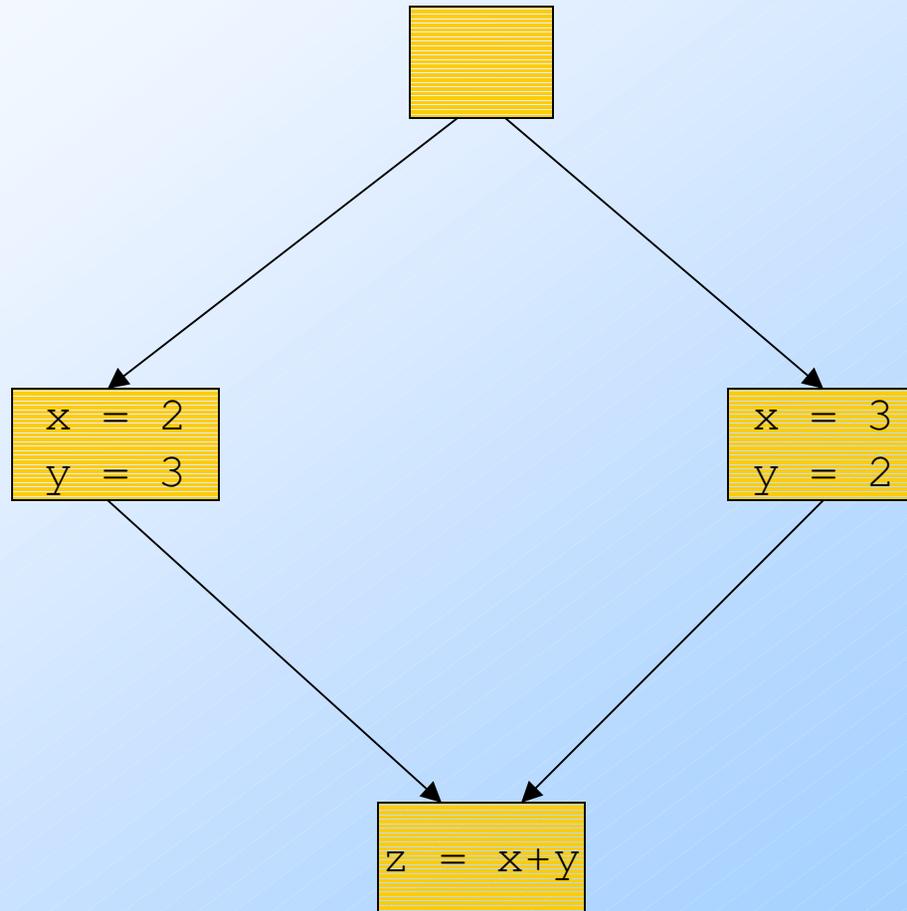
# Flow/Context Insensitivity

- ◆ Not so bad when program units are small (few assignments to any variable).
- ◆ **Example:** Java code often consists of many small methods.
  - ◆ **Remember:** you can distinguish variables by their full name, e.g., `class.method.block.identifier`.

# Context Sensitivity

- ◆ Can distinguish paths to a given point.
- ◆ **Example:** If we remembered paths, we would not have the problem in the constant-propagation framework where  $x+y = 5$  but neither  $x$  nor  $y$  is constant over all paths.

# The Example Again



# An Interprocedural Example

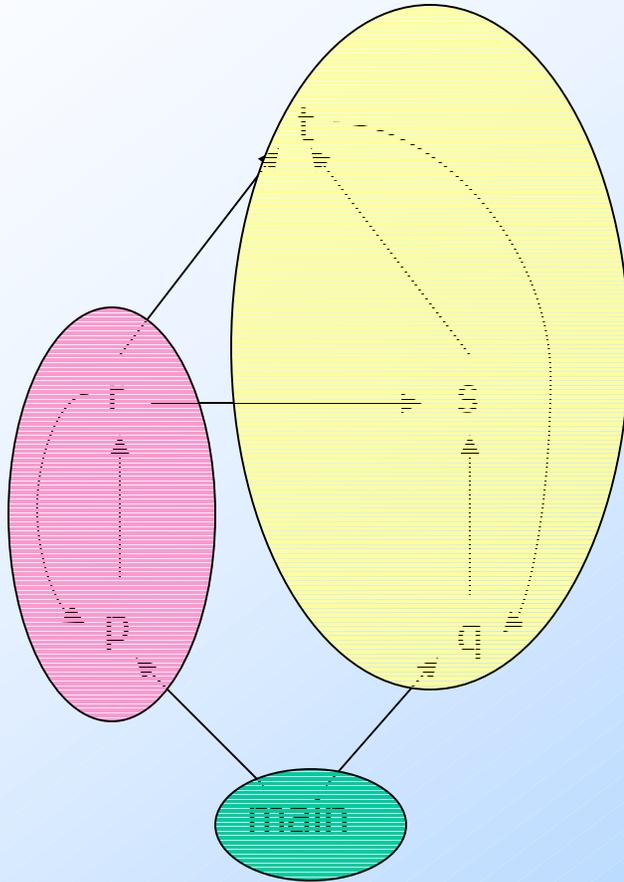
```
int id(int x) {return x;}  
void p() {a=2; b=id(a);...}  
void q() {c=3; d=id(c);...}
```

- ◆ If we distinguish **p** calling **id** from **q** calling **id**, then we can discover  $b=2$  and  $d=3$ .
- ◆ Otherwise, we think  $b, d = \{2, 3\}$ .

# Context-Sensitivity --- (2)

- ◆ Loops and recursive calls lead to an infinite number of contexts.
- ◆ Generally used only for interprocedural analysis, so forget about loops.
- ◆ Need to collapse strong components of the calling graph to a single group.
- ◆ “Context” becomes the sequence of groups on the calling stack.

# Example: Calling Graph



Contexts:

Green

Green, pink

Green, yellow

Green, pink, yellow

# Comparative Complexity

- ◆ **Insensitive**: proportional to size of program (number of variables).
- ◆ **Flow-Sensitive**: size of program, squared (points times variables).
- ◆ **Context-Sensitive**: worst-case exponential in program size (acyclic paths through the code).

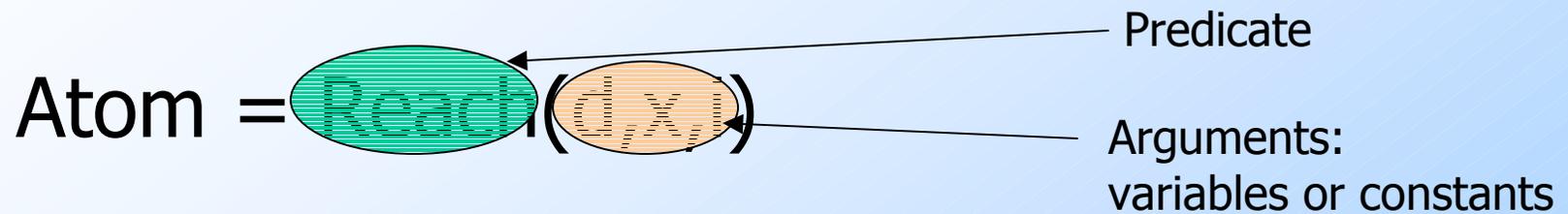
# Logical Representation

- ◆ We have used a set-theoretic formulation of DFA.
  - ◆  $IN$  = set of definitions, e.g.
- ◆ There has been recent success with a logical formulation, involving predicates.
- ◆ **Example:**  $Reach(d,x,i)$  = "definition  $d$  of variable  $x$  can reach point  $i$ ."

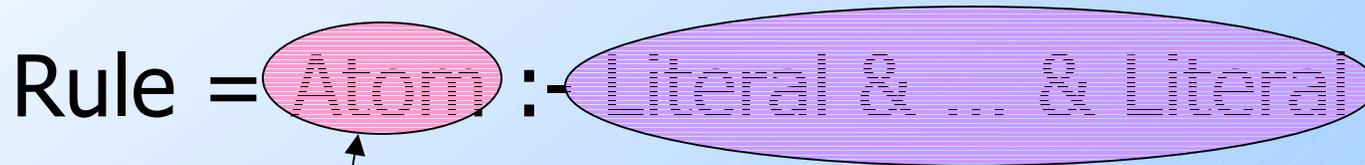
# Comparison: Sets Vs. Logic

- ◆ Both have an efficiency enhancement.
  - ◆ **Sets**: bit vectors and boolean ops.
  - ◆ **Logic**: BDD's, incremental evaluation.
- ◆ Logic allows integration of different aspects of a flow problem.
  - ◆ Think of PRE as an example. We needed 6 stages to compute what we wanted.

# Datalog --- (1)



Literal = Atom or NOT Atom



Make this  
atom true  
(the *head*).

The *body*:  
For each assignment of values  
to variables that makes all these  
true ...

# Example: Datalog Rules

Reach(d,x,j) :- Reach(d,x,i) &  
StatementAt(i,s) &  
NOT Assign(s,x) &  
Follows(i,j)

Reach(s,x,j) :- StatementAt(i,s) &  
Assign(s,x) &  
Follows(i,j)

# Datalog --- (2)

- ◆ **Intuition:** subgoals in the body are combined by “and” (strictly speaking: “join”).
- ◆ **Intuition:** Multiple rules for a predicate (head) are combined by “or.”

# Datalog --- (3)

- ◆ Predicates can be implemented by relations (as in a database).
- ◆ Each tuple, or assignment of values to the arguments, also represents a propositional (boolean) variable.

# EDB Vs. IDB Predicates

- ◆ Some predicates come from the program, and their tuples are computed by inspection.
  - ◆ Called *EDB*, or *extensional database* predicates.
- ◆ Others are defined by the rules only.
  - ◆ Called *IDB*, or *intensional database* predicates.

# Iterative Algorithm for Datalog

- ◆ Start with the EDB predicates = “whatever the code dictates,” and with all IDB predicates empty.
- ◆ Repeatedly examine the bodies of the rules, and see what new IDB facts can be discovered from the EDB and existing IDB facts.

# *Seminaive* Evaluation

- ◆ Remember that a **new** fact can be inferred by a rule in a given round only if it uses in the body some fact discovered on the previous round.
- ◆ Same idea applies to set-theoretic DFA, but the bit-vector implementation makes the idea ineffective.

# Example: Seminaive

$\text{Path}(x,y) \text{ :- Arc}(x,y)$

$\text{Path}(x,y) \text{ :- Path}(x,z) \ \& \ \text{Path}(z,y)$

$\text{NewPath}(x,y) = \text{Arc}(x,y); \text{Path}(x,y) = \emptyset;$

while ( $\text{NewPath} \neq \emptyset$ ) do {

$\text{NewPath}(x,y) = \{ (x,y) \mid \text{NewPath}(x,z)$

$\&\& \text{Path}(z,y) \mid \mid \text{Path}(x,z) \ \&\&$

$\text{NewPath}(z,y) \} - \text{Path}(x,y);$

$\text{Path}(x,y) = \text{Path}(x,y) \cup \text{NewPath}(x,y);$

}

# Stratification

- ◆ A risk occurs if there are negated literals involved in a recursive predicate.
  - ◆ Leads to oscillation in the result.
- ◆ Requirement for *stratification* :
  - ◆ Must be able to order the IDB predicates so that if a rule with P in the head has NOT Q in the body, then Q is either EDB or earlier in the order than P.

# Example: Nonstratification

$P(x) :- E(x) \ \& \ \text{NOT } P(x)$

- ◆ If  $E(1)$  is true, is  $P(1)$  true?
- ◆ It is after the first round.
- ◆ But not after the second.
- ◆ True after the third, not after the fourth,...

# Example: Stratification

- ◆ PRE is an example of stratified logic.
- ◆ Each of the analyses depends on previous ones, some negatively.
- ◆ But there is no recursion or iteration involving negation of the data-flow values we are trying to compute.

# PRE Example

Anticipated(B) :- (some rules)

Available(B) :- (some other rules)

Earliest(B) :- Anticipated(B) & **NOT** Available(B)

Postponable(B) :- (some rules involving Earliest)

Latest(B) :- (some rules involving Earliest,

Postponable, **NOT** Earliest,

and **NOT** Postponable)

Used(B) :- (rules involving Latest)

# New Topic: Pointer Analysis

- ◆ We shall consider Andersen's formulation of Java object references.
- ◆ Flow/context insensitive analysis.
- ◆ Cast of characters:
  1. **Local variables**, which point to:
  2. **Heap objects**, which may have fields that are references to other heap objects.

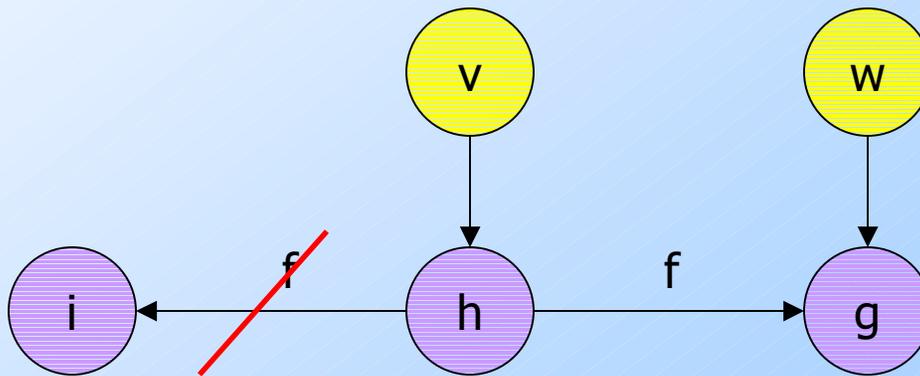
# Representing Heap Objects

- ◆ A heap object is named by the statement in which it is created.
- ◆ Note many run-time objects may have the same name.
- ◆ **Example:** `h: T v = new T;` says variable `v` can point to (one of) the heap object(s) created by statement `h`.



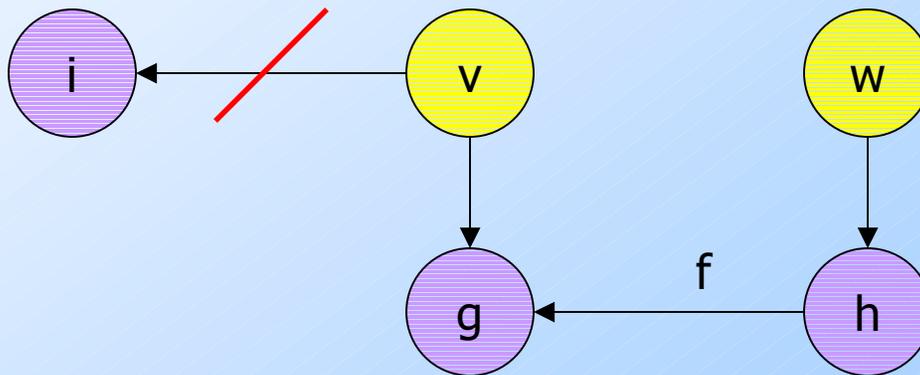
# Other Relevant Statements

- ◆  $v.f = w$  makes the  $f$  field of the heap object  $h$  pointed to by  $v$  point to what variable  $w$  points to.



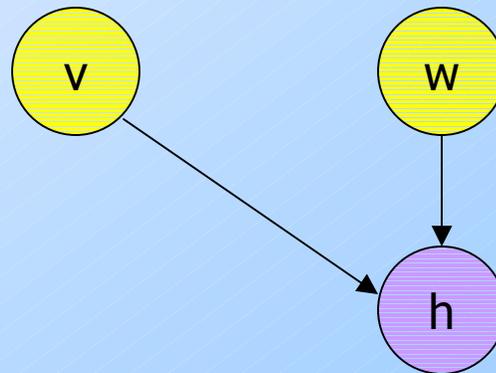
# Other Statements --- (2)

- ◆  $v = w.f$  makes  $v$  point to what the  $f$  field of the heap object  $h$  pointed to by  $w$  points to.



# Other Statements --- (3)

- ◆  $v = w$  makes  $v$  point to whatever  $w$  points to.
  - ◆ *Interprocedural Analysis* : Also models copying an actual parameter to the corresponding formal or return value to a variable.



# EDB Relations

- ◆ The facts about the statements in the program and what they do to pointers are accumulated and placed in several EDB relations.
- ◆ **Example:** there would be an EDB relation  $\text{Copy}(\text{To}, \text{From})$  whose tuples are the pairs  $(v, w)$  such that there is a copy statement  $v = w$ .

# Convention for EDB

- ◆ Instead of using EDB relations for the various statement forms, we shall simply use the quoted statement itself to stand for an atom derived from the statement.
- ◆ **Example:** “ $v=w$ ” stands for  $\text{Copy}(v,w)$ .

# IDB Relations

- ◆  $\text{Pts}(V, H)$  will get the set of pairs  $(v, h)$  such that variable  $v$  can point to heap object  $h$ .
- ◆  $\text{Hpts}(H1, F, H2)$  will get the set of triples  $(h, f, g)$  such that the field  $f$  of heap object  $h$  can point to heap object  $g$ .

# Datalog Rules

1.  $\text{Pts}(V,H) :- \text{"H: } V = \text{new T"}$
2.  $\text{Pts}(V,H) :- \text{"V=W"} \ \& \ \text{Pts}(W,H)$
3.  $\text{Pts}(V,H) :- \text{"V=W.F"} \ \& \ \text{Pts}(W,G) \ \& \ \text{Hpts}(G,F,H)$
4.  $\text{Hpts}(H,F,G) :- \text{"V.F=W"} \ \& \ \text{Pts}(V,H) \ \& \ \text{Pts}(W,G)$

# Example

```
T p(T x) {
    h: T a = new T;
    a.f = x;
    return a;
}
void main() {
    g: T b = new T;
    b = p(b);
    b = b.f;
}
```

# Apply Rules Recursively --- Round 1

```
T p(T x) {h: T a = new T;  
        a.f = x; return a;}
```

```
void main() {g: T b = new T;  
            b = p(b); b = b.f;}
```

Pts(a,h)

Pts(b,g)

# Apply Rules Recursively --- Round 2

```
T p(T x) {h: T a = new T;  
        a.f = x; return a;}  
void main() {g: T b = new T;  
            b = p(b); b = b.f;}
```

Pts(a,h)  
Pts(b,g)  
-----  
Pts(x,g)  
Pts(b,h)

# Apply Rules Recursively --- Round 3

```
T p(T x) {h: T a = new T;  
    a.f = x; return a;}  
void main() {g: T b = new T;  
    b = p(b); b = b.f;}
```

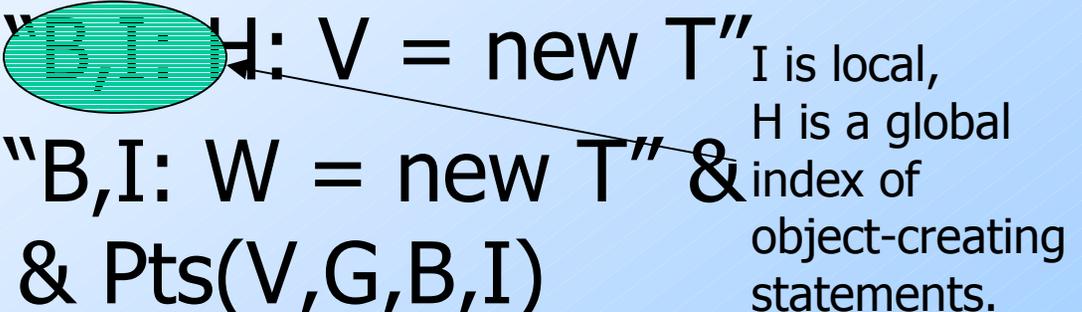
Pts(a,h)  
Pts(b,g)  
Pts(x,g)  
Pts(b,h)  
Pts(x,h)                      Hpts(h,f,g)



# Extension to Flow Sensitivity

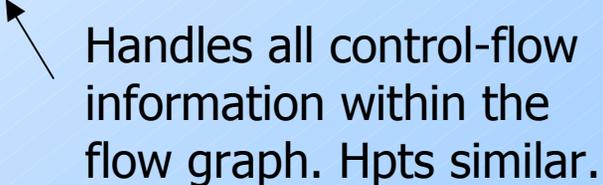
- ◆ IDB predicates need additional arguments  $B, I$ .
  - ◆  $B$  = block number.
  - ◆  $I$  = position within block,  $0, 1, \dots, n$  for  $n$ -statement block.
    - Position 0 is before first statement, position 1 is between 1<sup>st</sup> and 2<sup>nd</sup> statement, etc.

# Example of Rules: Flow Sensitive Pointer Analysis

$\text{Pts}(V, H, B, I+1) :- \text{"B, I: H: V = new T"}$   I is local,  
H is a global  
index of  
object-creating  
statements.

$\text{Pts}(V, G, B, I+1) :- \text{"B, I: W = new T"}$  &  
 $V != W$  &  $\text{Pts}(V, G, B, I)$

$\text{Pts}(V, G, B, I+1) :- \text{"B, I: W.f = X"}$  &  
 $\text{Pts}(V, G, B, I)$   Notice W=V OK

$\text{Pts}(V, G, B, 0) :- \text{Pts}(V, G, C, n)$  & "C is a  
predecessor block of B with  $n$  statements"  
 Handles all control-flow  
information within the  
flow graph. Hpts similar.

# Adding Context Sensitivity

- ◆ Include a component  $C = \text{context}$ .
  - ◆  $C$  doesn't change within a function.
  - ◆ Call and return can extend the context if the called function is not mutually recursive with the caller.

# Example of Rules: Context Sensitive

$\text{Pts}(V, H, B, I+1, C) :- \text{"B,I: } V=W\text{"} \&$   
 $\text{Pts}(W, H, B, I, C)$

$\text{Pts}(X, H, B0, 0, D) :- \text{Pts}(V, H, B, I, C) \&$   
 $\text{"B,I: call P(..., V, ...)" \&$   
 $\text{"X is the corresponding actual to V in P"}$   
 $\& \text{"B0 is the entry of P" \&}$   
 $\text{"context D is C extended by P"}$