**CS109A Notes for Lecture 2/28/96**

## Binary Trees

Every binary tree has two "slots" for children; it may have none, either one, or both.

- *Empty* (0-node) binary tree is possible.
- Equivalently, a node has left and right subtrees. Either or both may be null.

## Data Structure for Binary Trees

A node is a record structure; a binary tree is a pointer to a node.

- `NULL` pointer represents the empty tree; otherwise a tree is a pointer to its root node.
- Nodes have fields **`leftchild`**, **`rightchild`**.
  - ☐ These point to roots of left/right subtrees, (left/right children).
  - ☐ They are null if left/right subtree is `NULL` (left/right child does not exist).
- Other fields within nodes are possible, e.g., label, pointer to parent.

## Structural Induction on Binary Trees

- One important difference: basis is the empty tree, not a tree of one node.

**Example:**

- $S(T)$: In a binary tree $T$ represented by left- and right-child pointers, there is one more `NULL` pointer than node.

**Basis:** If $T$ is the empty tree, then there is a `NULL` pointer that represents the tree as a whole. There are no nodes, so $S(T)$ holds for $T =$ empty tree.

**Induction:** Let $T$ not be empty and have left and right subtrees $L$ and $R$.

- By the IH, $L$ and $R$ each have one more `NULL` than node. "Excess" $= 2$.

- However, $T$ also has its root node, so the excess for $T$ is 1, proving $S(T)$.

### Binary Search Trees

- Labels at nodes, ordered by some $<$ comparison operator, e.g., ints, reals, strings.

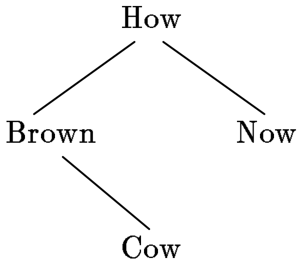
- If a node has label $x$, then every label in the right subtree is $> x$, and every label in the left subtree is $< x$.

- Supports *dictionary* = set with operations lookup, insert, delete.

  - ☐ Running time $= O(\log n)$ per operation on the average; $n$ = size of set.

- Supports *range queries* = find values between upper and lower limits.

**Example:**




### Lookup

- Key point: label at root tells us which half of the tree we must search, either left or right.

  - ☐ Thus, on the average, we cut the size of the tree to search almost in half in $O(1)$ time. After average $O(\log n)$ steps, we are down to 1 element and are done.

- Searching for $x$ at tree $T$:

**Basis:**

1. If $T$ is empty, fail; $x$ is not there.

2. If $T$ has label $x$ at the root, then found.

**Induction:** Let $T$ have root label $y$. If $x < y$, lookup $x$ on the left subtree of the root; if $x > y$ lookup $x$ on the right subtree.

### Insertion

Two approaches in C:

1. Insertion function gets tree (pointer to node) as argument and returns a revised tree including inserted element.

2. Insertion function gets pointer to tree (pointer to pointer to node) as argument and, when it needs to insert, creates a new node and makes the slot pointed to by its argument point to the new node.

- We'll sketch (1), typified by code in Fig. 5.35; (2) is typified by code of Fig. 5.38 for "delete."

**Basis:**

1. If $T$ is `NULL` create a new node with label $x$ and return a pointer to that node.

2. If $x$ at root, no action needed so return $T$.

**Induction:** Let root of $T$ have label $y$. If $x < y$, insert $x$ into left subtree. The left-subtree pointer at the root of $T$ becomes whatever tree is returned by recursive call. If $x > y$, do analogously at right.

### Deletion

To delete $x$ from tree $T$:

**Basis:** If $T$ is empty, just return $T$; if $x$ at root, delete root, fix up $T$ (explained next), and return the fixed-up $T$.

**Induction:** If $T$ has label $y$ at root, delete $x$ from left/right subtree if $x < y/x > y$. Replace left/right pointer by returned tree, and return the resulting $T$.

### Fixup (Deletemin)

- If we need to delete the root of $T$, if it has one `NULL` subtree, just return the other subtree (even if it too is `NULL` ).

- Otherwise, find the least element in the right subtree (by going down the leftmost path) and move it to the root of $T$.