CS109B Notes for Lecture 4/5/95

Graphs

- Nodes + edges = undirected graph.
- Nodes + arcs (= directed edges) = directed graph.
- Labels on nodes or arcs/edges are possible.

Paths

- In directed graph: sequence of nodes with arc from each node to the next.
- In undirected graph: sequence of nodes with an edge between each two consecutive nodes.
- *Length* of path = number of edges/arcs thereon.
- If edges/arcs are labeled by numbers, then we can sum the labels along a path to get a *distance*.

Cycles

- In directed graph: path that begins and ends at the same node.
  - □ *Simple* cycle: no repeats except the ends.
  - □ Note a cycle has many paths representing it, since the begin/end point may be any node on the cycle.
- In undirected graph: Simple cycle = sequence of 3 or more nodes with same begin/end point, but no other repetitions.
  - □ "Cycle" in undirected graph is tricky; see FCS, p. 456.

Adjacency List Representation

- Array or list of *headers*, one for each node.
- Undirected graph: header points to list of *adjacent* (shares an edge) nodes.

- Directed case: header for node v points to list of *successors* (nodes w with arc $v \to w$).
  - *Predecessor* = inverse of "successor."
- Labels for nodes may be attached to header for that node.
- Labels for arcs/edges are attached to the list cell for the "other" node.
  - Note an edge is represented twice.

Adjacency Matrices

- Node names must be integers `[0..MAX-1]`.
- `M[i][j] = TRUE` iff there is an edge between nodes i and j (arc $i \to j$ in directed case).
- Node labels in separate array.
- Edge/arc labels can be the value of `M[i][j]`.
  - Needs a special label that says "no edge/arc."

Size Parameters

We shall conventionally use:

- n = number of nodes of a graph.
- m = larger of number of nodes and edges/arcs.
  - Note: $m \geq n$.

Class Problem

A `hub` in an undirected graph is a node with an edge to every other node.

- How fast (as a function of n and m) can we find whether or not a graph has a hub?
  - If the graph is represented by adjacency lists? Adjacency matrix?
  - In the worst case? Average case? What is an "average case" anyway?