From RE's to Automata

1. NFA's with $\varepsilon$-transitions. ($\varepsilon$-NFA's).
2. RE's $\rightarrow$ $\varepsilon$-NFA's.
3. $\varepsilon$-NFA's $\rightarrow$ NFA's.

$\varepsilon$-NFA's

Allow transition on $\varepsilon$.

- $\varepsilon$ is invisible as far as the string labeling the part from start state to accepting state is concerned.

Example: $a^*b \mid b^*a$ is accepted by the following $\varepsilon$-NFA.

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RE to $\varepsilon$-NFA

Produce a special kind of $\varepsilon$-NFA:

- One start, one accepting state.
- At most 2 arcs out of any state.

Construction of $\varepsilon$-NFA from RE is a structural induction on the expression tree for the RE.

- See pp. 574–5, FCS for pictures.

Basis: Operand: $\emptyset$, $\varepsilon$, or a symbol $a$. 
**Induction:** Cases for |, concatenation, *.

- Inductive hypothesis \( S(R) \): the \( \epsilon \)-NFA constructed for RE \( R \) has paths from start to accepting state labeled by all and only the strings in \( L(R) \).

**\( \epsilon \)-NFA to NFA**

First step is to determine for all states \( s \) and \( t \) whether there is a path labeled \( \epsilon \) from \( s \) to \( t \).

- Special case of all-pairs shortest path: give \( \epsilon \)-arc a weight 0 and other arcs or no arc a weight \( \infty \).

□ Ask: is the distance from \( s \) to \( t \) 0?

**Example:** Here is the above \( \epsilon \)-NFA with non-\( \epsilon \) arcs removed.

Here are the reaching pairs:

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- **Important state** = start state or a state with a non-\( \epsilon \) transition in.

**Example:** For our running example, all but 6 are important.

- Eliminate \( \epsilon \)-transitions by:
If there is an ε-path from important state \( s \) to \( t \) and a transition on \( t \) to \( r \) on symbol \( a \) (therefore \( r \) is surely important), then add a transition from \( s \) to \( r \) on \( a \).

Important state \( s \) is accepting iff there is a (possibly empty) ε-path from \( s \) to an accepting state.

Example:

![Diagram](image_url)

**FA to RE**

Key idea: *pivot* on a state (like Floyd's algorithm).

- Picture, p. 583, FCS.
- Initially, label of a FA arc is treated as a RE.
- If we pivot on state \( u \), consider a predecessor state \( s \) and a successor state \( t \).
• New RE for going from \( s \) to \( t \) is \( R | S U^* T \).

Why?

Reducing the Automaton

If there is one accepting state, and it is not the start state, eliminate all other states.

• The result is a 2-state automaton with RE's on 4 arcs. Fig. 10.43, p. 586, FCS, gives the automaton and the resulting RE.

Some additional details:

• If start = accepting, you get a 1-state automaton as in Fig. 10.44.

• If there is more than 1 accepting state, repeat process for each and take the union of the resulting RE's.

Example:

Resulting RE: \( (00)^* 01 \left(11 | 10(00)^*01\right)^* \).