CS109A Notes for Lecture 1/31/96

Measuring the Running Time of Programs

Fix a measure of the "size" n of the data to which a program is being applied.

Example: For integer arguments, the value is often a good size measure. For strings: the length.

- Compute a big-oh upper bound on the running time of a program by induction on the *complexity* of program structures, i.e., the depth to which structures are nested.
  - □ Try to make the bound simple and tight.
- In the following, we assume there are no function calls in the program except for I/O operations.

Basis: *Simple statements* contain no statements nested within them. In C:

1. Assignment statements.
2. Goto's, including `break`, `continue`, `return`.
3. Input/Output using function calls like `printf` or `getchar()`.

- Fundamental assumption: Application of an operator takes a constant amount of time.
  - □ Write "some constant" as $O(1)$.
  - □ Operators include arithmetic, comparison, logical.
  - □ ML has some exceptions: concatenation of strings or lists.
- Thus, in C every simple statement takes $O(1)$ time.

Induction: Complex statements built from simple statements by recursive application of:

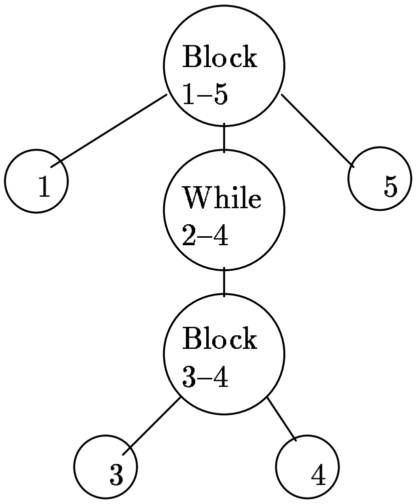
1. Loop formers: for-, while-, repeat-.
2. Branching statements: if$\cdots$else-, if-, case-.
3. Block formers: `{`$\cdots$`}`.

Structure Trees

- node = complex statement; its children are the constituent statements.

Example:

```
      main() {
          int i;
(1)       scanf("%d", &i);
(2)       while(i>0) {
(3)           putchar('0' + i%2);
(4)           i /= 2;
          }
(5)       putchar('\n');
      }
```



Details of the Induction

- *Blocks*: Running time bound = sum of the bounds of the constituents.
  - □ Use summation law to drop from the sum any term that is big-oh of another term.
- *Conditionals*: Bound = $O(1)$ + larger of bounds for the if- and else- parts.
  - □ $O(1)$ is for cost of the test — usually neglectable.
- *Loops*: Bound is usually the maximum number of times around the loop $\times$ the bound on the time to execute the loop body.

- ☐ But we must include $O(1)$ for the increment and test each time around the loop.
- ☐ The possibility that the loop is executed 0 times must be considered. Then, $O(1)$ for the initialization and first test is the total cost.

Example: Consider binary-conversion function. Size of data $= i$.

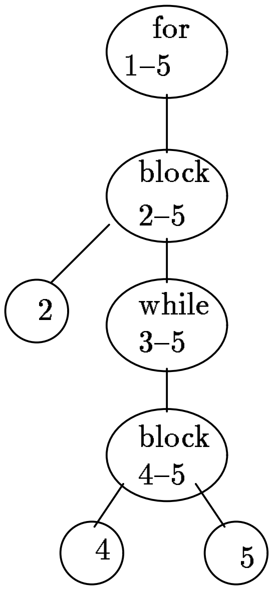
- Lines 1, 3, 4, 5 each $O(1)$ by the basis.
- Block of 3–4 is $O(1) + O(1) = O(1)$.
- While of 2–4 iterates at most $\log_2 i$ times. Bound on body times number of iterations $= O(1) \times \log_2 i = O(\log i)$.
- Block of 1–5 is $O(1) + O(\log i) + O(1) = O(\log i)$.
- i.e., it takes $O(\log i)$ time to convert i to binary by this function.

Triangular Double Loops

Sometimes we need to "give up" trying to tighten the upper bound on running time. Example: an inner loop iterates different numbers of times.

Example: *Insertion sort*: After i iterations, the first i elements of an array are sorted. At iteration $i + 1$ we move the $(i + 1)$st element forward until it meets an element smaller than it.

```
     void isort(int A[], int n) {
         int i,j;
(1)      for(i=1; i<n; i++) {
(2)          j = i;
(3)          while(j>0 && A[j-1]>A[j]) {
(4)              swap(j-1, j); /* exchange A[j-1] with A[j] */
(5)              j--;
             }
         }
     }
```



- Input "size" $= n =$ length of array A.
- Lines 2, 4, 5 are $O(1)$; note *swap* is short for 3 assignment statements.
- Block 4–5 is $O(1) + O(1) = O(1)$.
- While-loop 3–5 iterates at most i times, for $j = i$ down to $j = 1$.
  - □ But it may terminate earlier if $A[j-1] \geq A[j]$ ever holds.
  - □ Thus, $i \times O(1) = O(i)$ is an upper bound on lines 3–5.
- Block 2–5 takes time $O(1) + O(i) = O(i)$.
- For-loop 1–5 iterates $n - 1$ times. The body takes $O(i)$ time.
  - □ But i changes within the loop and makes no sense outside the loop, so we cannot say the for-loop takes $O(ni)$ time.
  - □ But $n \geq i$, so $O(n)$ is an upper bound on the while-loop of 3–5.
  - □ Then, the upper bound on the for-loop is $n \times O(n) = O(n^2)$.
- Nothing lost. If we summed the times for each iteration of the for-loop we would get $\sum_{i=1}^{n-1} O(i) = O\big(n(n-1)/2\big) = O(n^2)$.