Challenge Problem Set 2

May 2, 2010

1 Problem 1. (5 points)
As per the given conditions, the 2’s can only occur in the beginning of the sentence. So one of the possible representation is $2^*(1 + \epsilon)(01)^*(0 + \epsilon)$.

Grading - There are many equivalent regular expressions, all were considered. In case the expression was incorrect, we tried to mention either the illegal expressions generated or some legal expression missed.

2 Problem 2. (5 points)
First, we can convert the NFA $A$ to a DFA $B$ by using the subset construction, where $B = (Q, \Sigma, \delta, q_0, F)$. We have $L(A) = L(B)$. We then change the final states of $B$ to nonfinal states and nonfinal states of $B$ to final states.

The language $L(C)$ of the resulting DFA $C = (Q, \Sigma, \delta, q_0, Q - F)$ is the complement of $L(A)$.

$w$ is in $L(C)$ if and only if $\hat{\delta}(q_0, w)$ is in $Q - F$, which occurs if and only if $w$ is not in $L(B)$. Since $L(B) = L(A)$, $L(C) = L(A)$.

3 Problem 3. (5 points)
The idea is to construct a DFA that recognizes all those $w$ for which $ww$ is not recognizes by the given DFA $A$. If this DFA recognizes the empty language, the property holds, else it does not.

To construct such a DFA, we first consider the DFA’s $A_i$’s, one for each final state of $A$. Each $A_i$ is identical to $A$, except for the fact that their
starting state is different, and one of the final states of A. We therefore have one $A_i$ for each final state of A. We construct our objective DFA B, by taking the product DFA of A and all these $A_i$’s.

Let $q_i$’s be the final states of A, and $q_0$ the start state of A. The start state of B is defined as $(q_0, q_1, ..., q_m)$. A state $(q_i, p_1, p_2, ..., p_m)$ is defined as a final state of B if $q_i$ is a final state of A and $p_i$ is not a final state of A. Basically after reading an input w if A reaches $q_i$, it reaches $p_i$ after reading w from $q_i$, by the definition of the automata $A_i$’s.

**Error Codes**

- **3.1** : The given construction does not work. 3 points deducted.
- **3.2** : The solution involves going through all possible w for A. 4 points deducted
- **3.3** : The solution lacks minor details in the end. 1 point deducted