CS154 Midterm Examination

Monday, February 7, 2000, 3:15–4:30PM

Directions

- The exam is *open book*; any written materials may be used.
- Answer all 6 questions on the exam paper itself.
- The total number of points is 100.
- Do not forget to **sign the pledge** below.

I acknowledge and accept the honor code.

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Print your name here: ____

1	2	3	4
5	6		

Problem 1: (20 points) Let L be the language consisting of all strings of zero or more 0's followed by one or more 1's, followed by two or more 2's. For example, 001122, 122, and 0111122 are in L; 012 (too few 2's), and 0112122 (a 2 precedes a 1) are not.

a) Write a regular expression whose language is L.

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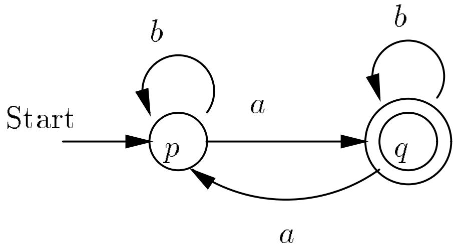
b) In the space below, draw the transition diagram of a DFA whose language is L. For full credit, your DFA should be as simple as possible. Note that constructing the DFA from your answer to (a) is not only not required, it will probably waste much time. Just reason out the DFA directly.

c) Give a context-free grammar generating the same language. It is sufficient just to give the productions, assuming that S is the start symbol.

Problem 2: (20 points) Here is the transition diagram for a simple DFA

$$A = (\{p, q\}, \{a, b\}, \delta, p, \{q\})$$



We are going to give you a proof that A accepts string w if and only if w has an odd number of a's. We will provide the argument; you have to give the reasons why each step is true. You should choose your reason by letter, from the following choices:

A. Well known facts about how integers and/or strings work, e.g., the sum of an odd number and an even number is odd, or the fact about strings that if $x = yz$, then the number of a's in x equals the sum of the number of a's in y plus the number of a's in z. These are not the only possible facts represented by choice A, just examples.
B. By definition of $\hat{\delta}$.
C. By the inductive hypothesis.

Place A, B, or C in each of the blanks below.

The proof is an induction on n, the length of w, that $\hat{\delta}(p, w) = q$ if and only if w has an odd number of a's.

Basis: If $n = 0$, then $w = \epsilon$. $\hat{\delta}(p, w) = p$, and w has zero a's, so the statement holds.

Induction: Assume that the statement holds for $n - 1$, and let w be of length n. There are two cases: $w = xa$ or $w = xb$, where x represents the first $n - 1$ positions of w. Consider first the case, that $w = xa$. If w has an even number of a's then x has an odd number of a's because ________. Thus, $\hat{\delta}(p, x) = q$ because ________. Therefore $\hat{\delta}(p, w) = p$ because ________. Now suppose w has an odd number of a's. Then x has an even number of a's, because ________. $\hat{\delta}(p, x) = p$ because ________. $\hat{\delta}(p, w) = q$ because ________. These statements complete the proof of the case $w = xa$.

Now, you should do the proof of the case $w = xb$ in your own words. You may continue to use A, B, and C as shorthands for reasons.

Problem 3: (15 points) If L is a language and a a symbol, then $a \backslash L = \{w \mid aw \text{ is in } L\}$, that is, the set of strings w such that aw is in L. We wish to prove that if L is regular then $a \backslash L$ is regular. To do so, we shall start with a DFA $A = (Q_A, \Sigma, \delta_A, q_A, F_A)$ for L (i.e., $L(A) = L$) and construct an ϵ-NFA $B = (Q_B, \Sigma, \delta_B, q_B, F_B)$ for $a \backslash L$. Note that B is **not** a DFA. We'll start you off, and you will provide much of the construction for B. Although this closure property looks something like HW2, Problem 6, the construction to be used here is quite different from the solution to that problem. You need not prove that your construction is correct, although we ask for some intuition below. To begin, $Q_B = Q_A \cup \{q_B\}$, where q_B is a new state, used as the start state for B.

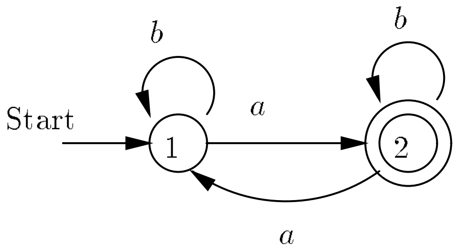
a) Describe the transitions out of the new state q_B.

b) What do you recommend for $\delta_B(q, b)$ if q is a state in Q_A and b is an input symbol in Σ?

c) What do you recommend as F_B?

d) Briefly explain why $L(B) = a\backslash L$.

Problem 4: (15 points) Here is the same DFA as for Question 2, with the states changed to 1 and 2.



We want to use the construction given in the reader to convert it to a regular expression. What are the following regular expressions? You may simplify your expressions, and do not have to use the recursive formula given in the reader (i.e., you may "reason out" what these expressions are).

a) $R_{11}^{(0)}$

b) $R_{22}^{(1)}$

c) $R_{12}^{(2)}$

Problem 5: (15 points) Prove that the language L consisting of all strings of 0's and 1's with more 0's than 1's is not a regular language. Note that there is no constraint on the order in which 0's and 1's appear in strings of L; the only constraint is on the numbers of 0's and 1's. Begin the proof by stating that if L is regular, then it has some pumping-lemma constant n.

a) What string do you choose for w?

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b) Suppose that the "adversary" breaks $w = xyz$, such that $|xy| \leq n$ and $y \neq \epsilon$. What value of i do you choose to create a string xy^iz that is not in L?________ Why is xy^iz not in L?

Problem 6: (15 points) Here is a context-free grammar $G = (\{S, A, B\}, \{0, 1\}, P, S)$, where P is the set of productions:

$$S \rightarrow 0A \mid 1B \mid \epsilon$$
$$A \rightarrow 1S \mid 0AA$$
$$B \rightarrow 0S \mid 1BB$$

Intuitively, A generates strings with one more 1 than 0, B generates strings with one more 0 than 1, and S generates the strings with equal numbers of 0's and 1's.

a) Give a leftmost derivation of the string 0011.

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b) Give a rightmost derivation of the string 001011.

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c) In the space below, draw a parse tree for the string 1001.