Mining of Massive Datasets

Jure Leskovec
Stanford Univ.

Anand Rajaraman
Milliway Labs

Jeffrey D. Ullman
Stanford Univ.

Preface

This book evolved from material developed over several years by Anand Rajaraman and Jeff Ullman for a one-quarter course at Stanford. The course CS345A, titled “Web Mining,” was designed as an advanced graduate course, although it has become accessible and interesting to advanced undergraduates. When Jure Leskovec joined the Stanford faculty, we reorganized the material considerably. He introduced a new course CS224W on network analysis and added material to CS345A, which was renumbered CS246. The three authors also introduced a large-scale data-mining project course, CS341. The book now contains material taught in all three courses.

What the Book Is About

At the highest level of description, this book is about data mining. However, it focuses on data mining of very large amounts of data, that is, data so large it does not fit in main memory. Because of the emphasis on size, many of our examples are about the Web or data derived from the Web. Further, the book takes an algorithmic point of view: data mining is about applying algorithms to data, rather than using data to “train” a machine-learning engine of some sort. The principal topics covered are:

1. Distributed file systems and map-reduce as a tool for creating parallel algorithms that succeed on very large amounts of data.
2. Similarity search, including the key techniques of minhashing and locality-sensitive hashing.
3. Data-stream processing and specialized algorithms for dealing with data that arrives so fast it must be processed immediately or lost.
4. The technology of search engines, including Google’s PageRank, link-spam detection, and the hubs-and-authorities approach.
5. Frequent-itemset mining, including association rules, market-baskets, the A-Priori Algorithm and its improvements.
6. Algorithms for clustering very large, high-dimensional datasets.
7. Two key problems for Web applications: managing advertising and recommendation systems.

8. Algorithms for analyzing and mining the structure of very large graphs, especially social-network graphs.

9. Techniques for obtaining the important properties of a large dataset by dimensionality reduction, including singular-value decomposition and latent semantic indexing.

10. Machine-learning algorithms that can be applied to very large data, such as perceptrons, support-vector machines, and gradient descent.

Prerequisites

To appreciate fully the material in this book, we recommend the following prerequisites:

1. An introduction to database systems, covering SQL and related programming systems.

2. A sophomore-level course in data structures, algorithms, and discrete math.

3. A sophomore-level course in software systems, software engineering, and programming languages.

Exercises

The book contains extensive exercises, with some for almost every section. We indicate harder exercises or parts of exercises with an exclamation point. The hardest exercises have a double exclamation point.

Support on the Web

Go to http://www.mmds.org for slides, homework assignments, project requirements, and exams from courses related to this book.

Gradiance Automated Homework

There are automated exercises based on this book, using the Gradiance root-question technology, available at www.gradiance.com/services. Students may enter a public class by creating an account at that site and entering the class with code 1EDD8A1D. Instructors may use the site by making an account there
and then emailing support at gradiance dot com with their login name, the name of their school, and a request to use the MMDS materials.

Acknowledgements

Cover art is by Scott Ullman.

We would like to thank Foto Afrati, Arun Marathe, and Rok Sosic for critical readings of a draft of this manuscript.

Errors were also reported by Rajiv Abraham, Ruslan Aduk, Apoorv Agarwal, Aris Anagnostopoulos, Yokila Arora, Atilla Soner Balkir, Arnaud Belletoiile, Robin Bennett, Susan Biancani, Amitabh Chaudhary, Leland Chen, Hua Feng, Marcus Gemeinder, Anastasios Gounaris, Clark Grubb, Shrey Gupta, Waleed Hameid, Saman Haratizadeh, Przemyslaw Horban, Jeff Hwang, Rafi Kamal, Lachlan Kang, Ed Knorr, Haewoon Kwak, Ellis Lau, Greg Lee, David Z. Liu, Ethan Lozano, Yuman Luo, Michael Mahoney, Justin Meyer, Bryant Moscon, Brad Penoff, John Phillips, Philips Kokoh Prasetyo, Qi Ge, Harizo Rajaona, Timon Ruban, Rich Seiter, Hitesh Shetty, Angad Singh, Sandeep Sripada, Dennis Sidharta, Krzysztof Stencel, Mark Storus, Roshan Sumbaly, Zack Taylor, Tim Triche Jr., Wang Bin, Weng Zhen-Bin, Robert West, Oscar Wu, Xie Ke, Nicolas Zhao, and Zhou Jingbo, The remaining errors are ours, of course.

J. L.
A. R.
J. D. U.
Palo Alto, CA
March, 2014
Contents

1 Data Mining 1
   1.1 What is Data Mining? 1
      1.1.1 Statistical Modeling 1
      1.1.2 Machine Learning 2
      1.1.3 Computational Approaches to Modeling 2
      1.1.4 Summarization 3
      1.1.5 Feature Extraction 4
   1.2 Statistical Limits on Data Mining 4
      1.2.1 Total Information Awareness 5
      1.2.2 Bonferroni’s Principle 5
      1.2.3 An Example of Bonferroni’s Principle 6
      1.2.4 Exercises for Section 1.2 7
   1.3 Things Useful to Know 7
      1.3.1 Importance of Words in Documents 7
      1.3.2 Hash Functions 9
      1.3.3 Indexes 10
      1.3.4 Secondary Storage 11
      1.3.5 The Base of Natural Logarithms 12
      1.3.6 Power Laws 13
      1.3.7 Exercises for Section 1.3 15
   1.4 Outline of the Book 15
   1.5 Summary of Chapter 1 17
   1.6 References for Chapter 1 18

2 MapReduce and the New Software Stack 21
   2.1 Distributed File Systems 22
      2.1.1 Physical Organization of Compute Nodes 22
      2.1.2 Large-Scale File-System Organization 23
   2.2 MapReduce 24
      2.2.1 The Map Tasks 25
      2.2.2 Grouping by Key 26
      2.2.3 The Reduce Tasks 27
      2.2.4 Combiners 27
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2.2</td>
<td>Choosing the Shingle Size</td>
<td>78</td>
</tr>
<tr>
<td>3.2.3</td>
<td>Hashing Shingles</td>
<td>79</td>
</tr>
<tr>
<td>3.2.4</td>
<td>Shingles Built from Words</td>
<td>79</td>
</tr>
<tr>
<td>3.2.5</td>
<td>Exercises for Section 3.2</td>
<td>80</td>
</tr>
<tr>
<td>3.3</td>
<td>Similarity-Preserving Summaries of Sets</td>
<td>80</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Matrix Representation of Sets</td>
<td>81</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Minhashing</td>
<td>81</td>
</tr>
<tr>
<td>3.3.3</td>
<td>Minhashing and Jaccard Similarity</td>
<td>82</td>
</tr>
<tr>
<td>3.3.4</td>
<td>Minhash Signatures</td>
<td>83</td>
</tr>
<tr>
<td>3.3.5</td>
<td>Computing Minhash Signatures</td>
<td>83</td>
</tr>
<tr>
<td>3.3.6</td>
<td>Exercises for Section 3.3</td>
<td>86</td>
</tr>
<tr>
<td>3.4</td>
<td>Locality-Sensitive Hashing for Documents</td>
<td>87</td>
</tr>
<tr>
<td>3.4.1</td>
<td>LSH for Minhash Signatures</td>
<td>88</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Analysis of the Banding Technique</td>
<td>89</td>
</tr>
<tr>
<td>3.4.3</td>
<td>Combining the Techniques</td>
<td>91</td>
</tr>
<tr>
<td>3.4.4</td>
<td>Exercises for Section 3.4</td>
<td>91</td>
</tr>
<tr>
<td>3.5</td>
<td>Distance Measures</td>
<td>92</td>
</tr>
<tr>
<td>3.5.1</td>
<td>Definition of a Distance Measure</td>
<td>92</td>
</tr>
<tr>
<td>3.5.2</td>
<td>Euclidean Distances</td>
<td>93</td>
</tr>
<tr>
<td>3.5.3</td>
<td>Jaccard Distance</td>
<td>94</td>
</tr>
<tr>
<td>3.5.4</td>
<td>Cosine Distance</td>
<td>95</td>
</tr>
<tr>
<td>3.5.5</td>
<td>Edit Distance</td>
<td>95</td>
</tr>
<tr>
<td>3.5.6</td>
<td>Hamming Distance</td>
<td>96</td>
</tr>
<tr>
<td>3.5.7</td>
<td>Exercises for Section 3.5</td>
<td>97</td>
</tr>
<tr>
<td>3.6</td>
<td>The Theory of Locality-Sensitive Functions</td>
<td>99</td>
</tr>
<tr>
<td>3.6.1</td>
<td>Locality-Sensitive Functions</td>
<td>99</td>
</tr>
<tr>
<td>3.6.2</td>
<td>Locality-Sensitive Families for Jaccard Distance</td>
<td>100</td>
</tr>
<tr>
<td>3.6.3</td>
<td>Amplifying a Locality-Sensitive Family</td>
<td>101</td>
</tr>
<tr>
<td>3.6.4</td>
<td>Exercises for Section 3.6</td>
<td>103</td>
</tr>
<tr>
<td>3.7</td>
<td>LSH Families for Other Distance Measures</td>
<td>104</td>
</tr>
<tr>
<td>3.7.1</td>
<td>LSH Families for Hamming Distance</td>
<td>104</td>
</tr>
<tr>
<td>3.7.2</td>
<td>Random Hyperplanes and the Cosine Distance</td>
<td>105</td>
</tr>
<tr>
<td>3.7.3</td>
<td>Sketches</td>
<td>106</td>
</tr>
<tr>
<td>3.7.4</td>
<td>LSH Families for Euclidean Distance</td>
<td>107</td>
</tr>
<tr>
<td>3.7.5</td>
<td>More LSH Families for Euclidean Spaces</td>
<td>108</td>
</tr>
<tr>
<td>3.7.6</td>
<td>Exercises for Section 3.7</td>
<td>109</td>
</tr>
<tr>
<td>3.8</td>
<td>Applications of Locality-Sensitive Hashing</td>
<td>110</td>
</tr>
<tr>
<td>3.8.1</td>
<td>Entity Resolution</td>
<td>110</td>
</tr>
<tr>
<td>3.8.2</td>
<td>An Entity-Resolution Example</td>
<td>111</td>
</tr>
<tr>
<td>3.8.3</td>
<td>Validating Record Matches</td>
<td>112</td>
</tr>
<tr>
<td>3.8.4</td>
<td>Matching Fingerprints</td>
<td>113</td>
</tr>
<tr>
<td>3.8.5</td>
<td>A LSH Family for Fingerprint Matching</td>
<td>114</td>
</tr>
<tr>
<td>3.8.6</td>
<td>Similar News Articles</td>
<td>115</td>
</tr>
<tr>
<td>3.8.7</td>
<td>Exercises for Section 3.8</td>
<td>117</td>
</tr>
<tr>
<td>3.9</td>
<td>Methods for High Degrees of Similarity</td>
<td>118</td>
</tr>
</tbody>
</table>
4 Mining Data Streams

4.1 The Stream Data Model ........................................... 131
  4.1.1 A Data-Stream-Management System .......................... 132
  4.1.2 Examples of Stream Sources ................................. 133
  4.1.3 Stream Queries ............................................. 134
  4.1.4 Issues in Stream Processing ................................. 135

4.2 Sampling Data in a Stream ....................................... 136
  4.2.1 A Motivating Example ....................................... 136
  4.2.2 Obtaining a Representative Sample ......................... 137
  4.2.3 The General Sampling Problem .............................. 137
  4.2.4 Varying the Sample Size .................................. 138
  4.2.5 Exercises for Section 4.2 ................................. 138

4.3 Filtering Streams ................................................ 139
  4.3.1 A Motivating Example ....................................... 139
  4.3.2 The Bloom Filter ........................................... 140
  4.3.3 Analysis of Bloom Filtering ................................. 140
  4.3.4 Exercises for Section 4.3 ................................. 141

4.4 Counting Distinct Elements in a Stream ....................... 142
  4.4.1 The Count-Distinct Problem ................................. 142
  4.4.2 The Flajolet-Martin Algorithm ............................. 143
  4.4.3 Combining Estimates ....................................... 144
  4.4.4 Space Requirements ....................................... 144
  4.4.5 Exercises for Section 4.4 ................................. 145

4.5 Estimating Moments .............................................. 145
  4.5.1 Definition of Moments ..................................... 145
  4.5.2 The Alon-Matias-Szegedy Algorithm for Second Moments .......................... 146
  4.5.3 Why the Alon-Matias-Szegedy Algorithm Works ............. 147
  4.5.4 Higher-Order Moments .................................... 148
  4.5.5 Dealing With Infinite Streams ............................ 148
  4.5.6 Exercises for Section 4.5 ................................. 149

4.6 Counting Ones in a Window ...................................... 150
  4.6.1 The Cost of Exact Counts ................................. 151
  4.6.2 The Datar-Gionis-Indyk-Motwani Algorithm ............... 151
  4.6.3 Storage Requirements for the DGIM Algorithm ............ 153
CONTENTS

4.6.4 Query Answering in the DGIM Algorithm . . . . . . . . . 153
4.6.5 Maintaining the DGIM Conditions . . . . . . . . . . . . 154
4.6.6 Reducing the Error . . . . . . . . . . . . . . . . . . . . . . 155
4.6.7 Extensions to the Counting of Ones . . . . . . . . . . . . 156
4.6.8 Exercises for Section 4.6 . . . . . . . . . . . . . . . . . . . 157
4.7 Decaying Windows . . . . . . . . . . . . . . . . . . . . . . . . . . 157
4.7.1 The Problem of Most-Common Elements . . . . . . . . . . 157
4.7.2 Definition of the Decaying Window . . . . . . . . . . . . 158
4.7.3 Finding the Most Popular Elements . . . . . . . . . . . . 159
4.8 Summary of Chapter 4 . . . . . . . . . . . . . . . . . . . . . . . 160
4.9 References for Chapter 4 . . . . . . . . . . . . . . . . . . . . . . 161

5  Link Analysis 163

5.1 PageRank . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 163
5.1.1 Early Search Engines and Term Spam . . . . . . . . . . . . 164
5.1.2 Definition of PageRank . . . . . . . . . . . . . . . . . . . . 165
5.1.3 Structure of the Web . . . . . . . . . . . . . . . . . . . . . 169
5.1.4 Avoiding Dead Ends . . . . . . . . . . . . . . . . . . . . . . 170
5.1.5 Spider Traps and Taxation . . . . . . . . . . . . . . . . . . 173
5.1.6 Using PageRank in a Search Engine . . . . . . . . . . . . . 175
5.1.7 Exercises for Section 5.1 . . . . . . . . . . . . . . . . . . . . 175
5.2 Efficient Computation of PageRank . . . . . . . . . . . . . . . . 177
5.2.1 Representing Transition Matrices . . . . . . . . . . . . . . 178
5.2.2 PageRank Iteration Using MapReduce . . . . . . . . . . . . 179
5.2.3 Use of Combiners to Consolidate the Result Vector . . . . . 179
5.2.4 Representing Blocks of the Transition Matrix . . . . . . . 180
5.2.5 Other Efficient Approaches to PageRank Iteration . . . . . 181
5.2.6 Exercises for Section 5.2 . . . . . . . . . . . . . . . . . . . . 183
5.3 Topic-Sensitive PageRank . . . . . . . . . . . . . . . . . . . . . . 183
5.3.1 Motivation for Topic-Sensitive Page Rank . . . . . . . . . . 183
5.3.2 Biased Random Walks . . . . . . . . . . . . . . . . . . . . . 184
5.3.3 Using Topic-Sensitive PageRank . . . . . . . . . . . . . . 185
5.3.4 Inferring Topics from Words . . . . . . . . . . . . . . . . . 186
5.3.5 Exercises for Section 5.3 . . . . . . . . . . . . . . . . . . . . 187
5.4 Link Spam . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 187
5.4.1 Architecture of a Spam Farm . . . . . . . . . . . . . . . . . . 187
5.4.2 Analysis of a Spam Farm . . . . . . . . . . . . . . . . . . . 189
5.4.3 Combating Link Spam . . . . . . . . . . . . . . . . . . . . . 190
5.4.4 TrustRank . . . . . . . . . . . . . . . . . . . . . . . . . . . . 190
5.4.5 Spam Mass . . . . . . . . . . . . . . . . . . . . . . . . . . . 191
5.4.6 Exercises for Section 5.4 . . . . . . . . . . . . . . . . . . . . 191
5.5 Hubs and Authorities . . . . . . . . . . . . . . . . . . . . . . . . . 192
5.5.1 The Intuition Behind HITS . . . . . . . . . . . . . . . . . . . 192
5.5.2 Formalizing Hubbiness and Authority . . . . . . . . . . . . 193
5.5.3 Exercises for Section 5.5 . . . . . . . . . . . . . . . . . . . . 196
6 Frequent Itemsets

6.1 The Market-Basket Model

6.1.1 Definition of Frequent Itemsets
6.1.2 Applications of Frequent Itemsets
6.1.3 Association Rules
6.1.4 Finding Association Rules with High Confidence
6.1.5 Exercises for Section 6.1

6.2 Market Baskets and the A-Priori Algorithm

6.2.1 Representation of Market-Basket Data
6.2.2 Use of Main Memory for Itemset Counting
6.2.3 Monotonicity of Itemsets
6.2.4 Tyranny of Counting Pairs
6.2.5 The A-Priori Algorithm
6.2.6 A-Priori for All Frequent Itemsets
6.2.7 Exercises for Section 6.2

6.3 Handling Larger Datasets in Main Memory

6.3.1 The Algorithm of Park, Chen, and Yu
6.3.2 The Multistage Algorithm
6.3.3 The Multihash Algorithm
6.3.4 Exercises for Section 6.3

6.4 Limited-Pass Algorithms

6.4.1 The Simple, Randomized Algorithm
6.4.2 Avoiding Errors in Sampling Algorithms
6.4.3 The Algorithm of Savasere, Omiecinski, and Navathe
6.4.4 The SON Algorithm and MapReduce
6.4.5 Toivonen's Algorithm
6.4.6 Why Toivonen's Algorithm Works
6.4.7 Exercises for Section 6.4

6.5 Counting Frequent Items in a Stream

6.5.1 Sampling Methods for Streams
6.5.2 Frequent Itemsets in Decaying Windows
6.5.3 Hybrid Methods
6.5.4 Exercises for Section 6.5

6.6 Summary of Chapter 6
6.7 References for Chapter 6

7 Clustering

7.1 Introduction to Clustering Techniques

7.1.1 Points, Spaces, and Distances
7.1.2 Clustering Strategies
7.1.3 The Curse of Dimensionality
8.2.4 Exercises for Section 8.2 ........................................ 286
8.3 The Matching Problem ............................................. 287
8.3.1 Matches and Perfect Matches ................................. 287
8.3.2 The Greedy Algorithm for Maximal Matching ............. 288
8.3.3 Competitive Ratio for Greedy Matching .................. 289
8.3.4 Exercises for Section 8.3 ................................. 290
8.4 The Adwords Problem ........................................... 290
8.4.1 History of Search Advertising .............................. 291
8.4.2 Definition of the Adwords Problem ....................... 291
8.4.3 The Greedy Approach to the Adwords Problem ........... 292
8.4.4 The Balance Algorithm .................................. 293
8.4.5 A Lower Bound on Competitive Ratio for Balance ...... 294
8.4.6 The Balance Algorithm with Many Bidders ............... 296
8.4.7 The Generalized Balance Algorithm ..................... 297
8.4.8 Final Observations About the Adwords Problem ......... 298
8.4.9 Exercises for Section 8.4 ................................. 299
8.5 Adwords Implementation ................................... 299
8.5.1 Matching Bids and Search Queries ....................... 300
8.5.2 More Complex Matching Problems ...................... 300
8.5.3 A Matching Algorithm for Documents and Bids ........ 301
8.6 Summary of Chapter 8 ........................................ 303
8.7 References for Chapter 8 .................................... 305

9 Recommendation Systems ......................................... 307
9.1 A Model for Recommendation Systems .................... 307
9.1.1 The Utility Matrix .................................... 308
9.1.2 The Long Tail ....................................... 309
9.1.3 Applications of Recommendation Systems ............. 309
9.1.4 Populating the Utility Matrix ......................... 311
9.2 Content-Based Recommendations .............................. 312
9.2.1 Item Profiles ...................................... 312
9.2.2 Discovering Features of Documents .................... 313
9.2.3 Obtaining Item Features From Tags .................... 314
9.2.4 Representing Item Profiles .............................. 315
9.2.5 User Profiles ....................................... 316
9.2.6 Recommending Items to Users Based on Content ....... 317
9.2.7 Classification Algorithms .............................. 318
9.2.8 Exercises for Section 9.2 ............................... 320
9.3 Collaborative Filtering .................................... 321
9.3.1 Measuring Similarity .................................. 322
9.3.2 The Duality of Similarity ................................ 324
9.3.3 Clustering Users and Items ............................ 325
9.3.4 Exercises for Section 9.3 ............................... 327
9.4 Dimensionality Reduction .................................. 328
9.4.1 UV-Decomposition .................................... 328
## 9.4.2 Root-Mean-Square Error ................................. 329
## 9.4.3 Incremental Computation of a UV-Decomposition .... 330
## 9.4.4 Optimizing an Arbitrary Element ...................... 332
## 9.4.5 Building a Complete UV-Decomposition Algorithm .... 334
## 9.4.6 Exercises for Section 9.4 ............................. 336

## 9.5 The Netflix Challenge ................................. 337

## 9.6 Summary of Chapter 9 ................................. 338

## 9.7 References for Chapter 9 ............................. 340

### 10 Mining Social-Network Graphs 343

#### 10.1 Social Networks as Graphs ............................ 343
  10.1.1 What is a Social Network? ............................ 344
  10.1.2 Social Networks as Graphs ............................ 344
  10.1.3 Varieties of Social Networks .......................... 346
  10.1.4 Graphs With Several Node Types .................... 347
  10.1.5 Exercises for Section 10.1 .......................... 348

#### 10.2 Clustering of Social-Network Graphs .................. 349
  10.2.1 Distance Measures for Social-Network Graphs ....... 349
  10.2.2 Applying Standard Clustering Methods ............... 349
  10.2.3 Betweenness ........................................ 351
  10.2.4 The Girvan-Newman Algorithm ....................... 351
  10.2.5 Using Betweenness to Find Communities ............. 354
  10.2.6 Exercises for Section 10.2 .......................... 356

#### 10.3 Direct Discovery of Communities ..................... 357
  10.3.1 Finding Cliques .................................... 357
  10.3.2 Complete Bipartite Graphs .......................... 357
  10.3.3 Finding Complete Bipartite Subgraphs ............... 358
  10.3.4 Why Complete Bipartite Graphs Must Exist .......... 359
  10.3.5 Exercises for Section 10.3 .......................... 361

#### 10.4 Partitioning of Graphs ............................... 361
  10.4.1 What Makes a Good Partition? ....................... 362
  10.4.2 Normalized Cuts .................................... 362
  10.4.3 Some Matrices That Describe Graphs ................. 363
  10.4.4 Eigenvalues of the Laplacian Matrix ................ 364
  10.4.5 Alternative Partitioning Methods ................... 367
  10.4.6 Exercises for Section 10.4 .......................... 368

#### 10.5 Finding Overlapping Communities ..................... 369
  10.5.1 The Nature of Communities .......................... 369
  10.5.2 Maximum-Likelihood Estimation ...................... 369
  10.5.3 The Affiliation-Graph Model ......................... 371
  10.5.4 Avoiding the Use of Discrete Membership Changes ... 374
  10.5.5 Exercises for Section 10.5 .......................... 375

#### 10.6 Simrank ............................................. 376
  10.6.1 Random Walkers on a Social Graph ................... 376
  10.6.2 Random Walks with Restart .......................... 377
10.6.3 Exercises for Section 10.6 ........................................... 380
10.7 Counting Triangles ..................................................... 380
10.7.1 Why Count Triangles? .............................................. 380
10.7.2 An Algorithm for Finding Triangles .............................. 381
10.7.3 Optimality of the Triangle-Finding Algorithm ................. 382
10.7.4 Finding Triangles Using MapReduce ............................. 383
10.7.5 Using Fewer Reduce Tasks ....................................... 384
10.7.6 Exercises for Section 10.7 ....................................... 385
10.8 Neighborhood Properties of Graphs ................................ 386
10.8.1 Directed Graphs and Neighborhoods ............................. 386
10.8.2 The Diameter of a Graph ......................................... 388
10.8.3 Transitive Closure and Reachability ............................ 389
10.8.4 Transitive Closure Via MapReduce ............................... 390
10.8.5 Smart Transitive Closure ......................................... 392
10.8.6 Transitive Closure by Graph Reduction ......................... 393
10.8.7 Approximating the Sizes of Neighborhoods .................... 395
10.8.8 Exercises for Section 10.8 ....................................... 397
10.9 Summary of Chapter 10 .............................................. 398
10.10 References for Chapter 10 ........................................... 402

11 Dimensionality Reduction ............................................. 405
11.1 Eigenvalues and Eigenvectors of Symmetric Matrices .......... 406
11.1.1 Definitions ......................................................... 406
11.1.2 Computing Eigenvalues and Eigenvectors ....................... 407
11.1.3 Finding Eigenpairs by Power Iteration ......................... 408
11.1.4 The Matrix of Eigenvectors ..................................... 411
11.1.5 Exercises for Section 11.1 ...................................... 411
11.2 Principal-Component Analysis ..................................... 412
11.2.1 An Illustrative Example ......................................... 413
11.2.2 Using Eigenvectors for Dimensionality Reduction .......... 416
11.2.3 The Matrix of Distances ......................................... 417
11.2.4 Exercises for Section 11.2 ...................................... 418
11.3 Singular-Value Decomposition ...................................... 418
11.3.1 Definition of SVD ............................................... 418
11.3.2 Interpretation of SVD ........................................... 420
11.3.3 Dimensionality Reduction Using SVD ......................... 422
11.3.4 Why Zeroing Low Singular Values Works ..................... 423
11.3.5 Querying Using Concepts ....................................... 425
11.3.6 Computing the SVD of a Matrix ................................. 426
11.3.7 Exercises for Section 11.3 ...................................... 427
11.4 CUR Decomposition .................................................. 428
11.4.1 Definition of CUR ............................................... 429
11.4.2 Choosing Rows and Columns Properly ......................... 430
11.4.3 Constructing the Middle Matrix ................................. 431
11.4.4 The Complete CUR Decomposition .............................. 432